ASSESSING STUDENTS’ MATHEMATICAL PROBLEM-SOLVING ABILITY

Sufyani Prabawanto¹, Nadya Syifa Utami², Aneu Pebrianti³

¹,²,³Departemen Pendidikan Matematika, Universitas Pendidikan Indonesia, Bandung, Indonesia

*Corresponding author.
E-mail: sufyani@upi.edu (¹)
nadyasyifa@upi.edu (²)
anupebrianti@upi.edu (³)

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Abstract

Students' ability to solve mathematical problems is the most important part of learning mathematics. Therefore, assessing students' ability to solve mathematical problems is one of the tasks of mathematics teachers that must be done. This study aims to develop a test instrument to assess students' ability to solve mathematical problems. The research method used Systematic Literature Review (SLR) by analyzing primary sources from books and articles. Before developing an instrument to assess students' ability to solve mathematical problems, the variables involved in mathematical problem solving were first analyzed. Furthermore, there are four domains involved in the instrument development framework carried out. They are resources, heuristics, control, and belief system. Based on this study, it was found that: (1) the aspects correlated with mathematical problem solving are reasoning, decision making, critical thinking, and creative thinking; (2) the types of tasks related to mathematical problem solving can be viewed from the structure, the number of essential steps needed to reach the solution, its orientation, and its presentation.

Keywords: Assessing problem solving ability; mathematical problem solving; test instruments.

INTRODUCTION

Mathematical problem solving is a very important ability and as a direction for students in learning mathematics. This problem-solving ability includes the ability to interpret problems well, being able to make mathematical designs/models, being able to provide alternative solutions based on the models that have been...
made. The urgency of mathematical problem-solving ability can also be seen from its existence as one of the process standards (Maf'Ulah et al., 2019). When compared with traditional materials, problem solving is more likely to increase student success with non-trivial tasks, interpretation of mathematical representations, and understanding of concepts, and does not rule out performance on basic skills. The description above shows that involvement in problem-solving activities can help students learn mathematics.

Furthermore, there is an argument about problem-solving based on mathematical concepts as “dynamic, problem-driven” (Liljedahl & Cai, 2021). From this perspective, Liljedahl & Cai states that mathematics as posing, refining and solving problem processes, not just a collection of finished products. Suseelan provides encouragement for problem solving that solving problems involves higher-order thinking skills (Suseelan et al., 2023). The final reason for problem solving in school mathematics is dealing with the relationship of the school curriculum and student daily life after school (Monica et al., 2019). They indicated that there is a gap between students' low-level problem-solving ability emphasized in test-driven curriculum topics and the kinds of understandings and abilities required to be successful outside of school. On the other hand, problem solving encourages the emergence of creativity, flexibility, and metacognitive thinking that is carried out according to professional needs and needs in everyday life. In other words, by learning problem solving, students have more opportunities to prepare themselves to face various aspects of their lives after finishing school.

For this reason, schools have been asked to present mathematical problem-solving as core of learning and teaching mathematics (Ramdan et al., 2018; Lestari et al., 2021). However, the practice that occurs in the classroom still tends to show completion procedures, work on some examples from the front of the class, and then give practice to students to follow the ways the teacher has shown in front of the class. Because, providing students with facts and procedures will not result in students who can effectively solve problems.

The frameworks commonly used in investigating mathematical problem-solving performance are: (1) resources, (2) heuristics, (3) control, and (4) belief system (Collins et al., 2018). Usually, resources are considered as the decisive factor in solving problems. This means, if the students’ prerequisite knowledge for solving the problems is available then the problem faced should be solved. However, sources of mathematical knowledge are a necessary condition but not sufficient for success in solving problems. This study will reveal the variables related to solving mathematical non-routine and develop instruments to assess students' mathematical problem-solving ability. The variables involved in this study include all aspects: (1) resources, (2) heuristics, (3) control, and (4) belief system, which has not been found in previous research. So, hopefully able to present a comprehensive alternative solution in reviewing students' problem-solving ability.

**METHOD**

This study was library research or sistematic literature review, especially books and journal articles. The steps used in this research consisted of
selecting the topic, identifying key words that describe this topic, searching information on the background, using online catalog to find relevant sources, using research database to find journal articles, evaluating the authority and quality of the material, and reviewing the progress.

The population in this study were articles and books that discuss problem solving assessment. And then, the data analysis technique be used to review the sources that had been collected, used descriptive qualitative.

RESULT AND DISCUSSION
The research results consist of two main issues, namely understanding mathematical problems and test instrument development.

Understanding Mathematical Problems
Learning mathematics should place problem-solving at the center. Before discussing further about solving this mathematical problem, it is necessary to clarify the meaning of the term "problem". Mathematics experts and teachers still do not agree on the meaning of the term "problem". For example, Polya distinguishes between authentic problems and routine problems. A task that can be completed by substituting certain facts into the starting situation is referred to as a routine problem, or a task that can be solve by applying certain procedures step by step, without tracing the originality of the problem. On the other hand, authentic problem is a task in which the method employed to get the solution is not known beforehand (Cho, et al., 2015). The same thing that problems divided into two types, namely routine problems and non-routine problems (Kablan & Uğur, 2021). From the division of this type of problem, implicitly, both Polya and Kablan & Uğur provide an understanding that a problem is something that must be resolved, ignoring the constraint factor, namely the way to solve it is not yet known.

A problem occurs when a person is faced with a situation that must be resolved but he does not know how to transform the situation towards his desired goal. In a monograph, in problem-solving, Gestalt's Karl Duncker defines that a problem arises when the steps to reach a desired goal are unknown. For one to move from the problem situation to the desired situation (solution), it requires one to think of alternatives. Therefore, one must be able to synthesize several alternatives in order to arrive at the desired situation. A situation is called a problem if there is no similar method to complete the task. A person is faced with a problem when he met a question and it cannot be answered or a situation where he/she cannot solve it immediately using the knowledge available to him. Kantowski also distinguishes problems (problem) and exercises (exercise). If in the "problem" the algorithm that students have cannot be used immediately or even the algorithm is not available, then in the "practice" the algorithms that exist in students can be used directly to solve the problem. Meanwhile, Schoenfeld (2016) stated that a problem can be defined as questions or situations that is challenging to manage and has never been solved. Fanani (2018) stated that finding solutions to "non-routine" mathematical problems requires strategies that do not only simply follow pre-existing procedures or algorithms, but also requires reasoning, argumentation and creativity in solving it. Barak & Assal (2018) stated that a problem is defined as a situation with
open-ended questions that intellectually challenge a person and support the development of procedures or algorithms to solve the questions. Thus, a problem is relative to the individuals involved; this means that one problem for one person may become not any problem for another. For example, $3 + 4$ may be a problem for preschoolers but not for junior high school students. Some definitions above make reference to the problem situation's starting statement (initial state) and objective state (goal state) in a problem situation.

The essence of the definitions of the problem (some other experts give the term non-routine problem) above is that a problem is a task or a situation faced by an individual for the first time and, thus, there is no ready procedure to solve it. The individual has to design his own solution method that describes the various skills, knowledge, strategies, etc., which he has learned. Thus, what makes something a problem or not a problem depends on the knowledge of the person dealing with it because the same task can be a no problem for one person and a problem for another. For example, the following problem is probably a problem for most junior high school students. How long will it take a water hyacinth to cover half of a lake if it doubles in size in a lake in 24 hours and covers the entire lake in 60 days? Problems like this are a type of problem he calls insight problems because students (problem solvers) are required to be able to find a method of solving them.

A routine problem-solving emphasizes the use of known procedures or algorithms to solve it. A routine problem with one, two, or many solving steps can be easily assessed using paper and pencil tests, especially those that focus on the use of algorithms. May be that's why routine problems are used by teachers widely. What students do in working process towards the completion of a mathematical task should refer to math problem-solving, which emphasizes the process involved, not just the final answer. From this perspective, some routine word-problems that appear in textbooks are not designed as mathematical problems. The questions are only as exercises (exercise). Problem solving emphasizes the use of heuristics. Heuristic is a strategy to obtain a solution to a problem. This strategy is general in nature so it does not guarantee the finding of a solution to a problem. Some examples of heuristics in solving mathematical problems include finding patterns, creating tables, building models, using pictures, simplifying problems, and working backwards.

Test Instrument Development

The development of mathematical problem-solving competency is a process cumulatively that depends on the student's learning experience about mathematical problem solving. Kilpatrick reveals that students' mathematical problem-solving ability is influenced by three main variables. The three main variables are subject variables, task variables, and situation variables (Verschaffel et al., 2020). In every mathematical problem-solving activity, the three main variables do not stand alone separately, but they are interactively involved and greatly determine the success or failure of students in getting the solution of mathematical problems.

Subject variables can be seen as variables that describe or measure the specific attributes of the subject, in this case students. Kilpatrick classifies these
subject variables based on how easy they are to modify or manipulate. Subject variables that are not open or difficult to modify are organismic variables. Age, gender, socioeconomic level, and geography are some examples of organismic factors. Kilpatrick noted that with the exception of sex, few studies have considered other organismal variables compared to describing the sample. Trait variables are subject variables that can be modified through processes, such as learning. Examples for trait variables are cognitive style, behavior, assertiveness, mathematical recall, or competence to predict the solution of a mathematical problem. Finally, the instructional history variables are variables that describe the school, the mathematical topics learned, or the problem-solving learning accepted by the subjects. Some of these variables are more open to be manipulated than others.

Kilpatrick cautions that failure to consider these variables in selecting the experimental group could be partly responsible for the weak differences found between learning methods. The success or failure of students in solving a problem is also determined by mental set. Mental set is some one’s tendency to solve certain problems in a fixed way based on previous situations for similar tasks. For example, a junior high school student is faced with the problem, "If a water hyacinth covers a lake twice as deep" 24 hours, and the whole lake was covered on the 60th day, on what day did the water hyacinth cover half the lake?" The student may fail to get the answer because he always thinks "work forward" in solving problems.

In the largest context, the social, psychological, or physical setting in which problem solving takes occurs is referred to as the situation. Several situational variables can coexist utilizing specific task variables, Kilpatrick refers to as format variables. The physical environment can include the kind of place (classroom, laboratory, outside the classroom, etc.), the character of the place (pleasant, exciting, and cozy, etc.), and the availability of resources (calculators, measuring materials, manipulative items, etc.). Psychological arrangements include anything that describes the objective of a situation (test, learning, practice, etc.), the method of action (investigative, prescriptive, and evaluative, etc.) and environment's suitability for learning (type or amount of benefit, quality, or quantity interaction). These situational variables are closely related to the subject's motivation in solving a problem.

Situational variables do not emphasize task or subject descriptions, but outside of both. These situational variables often receive little attention from researchers, they may have unexpected effects on problem solving performance. The significance of situation variables in problem solving was stated by Klein. He stated that to get to the solution of the sub-goal it is necessary to understand the situation, and those sub-goals influence his/her understanding positively or negatively (Klein & Crandall, 2018).

Problem solving is related to other concepts like reasoning, decision-making, critical and creative thinking, and thinking. Thinking describes the cognitive processes of the subject (as problem solvers). Thinking can be classified into two categories based on how it works: direct thinking, such as solving problems, and indirect thinking, like daydreaming. Because of this, problem solving is a subset of thinking,
namely direct thinking, which is a wider activity than problem solving.

Problem solving includes reasoning, decision-making, critical thinking, and creative thinking (Rahman, 2019). Making a conclusion from premises using logical rules by deduction or induction is one of the tasks that fall under the category of reasoning, which is an aspect of problem solving. For instance, students can deduce that all squares are quadrilaterals if they are aware that all shapes with four sides are quadrilaterals and that all squares have four sides. A learner can infer that the following number should be 10 if given a series of the digits 2, 4, 6, and 8. Making decisions is a component of problem solving and refers to particular tasks with the objective of selecting one of two or more choices depending on a number of factors. A decision-making exercise might ask a person to choose between earning a fixed $1,000 or having a 1% chance of winning $100,000. Therefore, issue solving, which is distinguished by its specialized duties, includes thinking and decision-making.

Certain aspects of problem solving are referred to as creative thinking and critical thinking. Creative thinking can be seen as ability to build alternative ways that match the criteria to reach a solution, while critical thinking can be seen as evaluating how well the various alternatives fit the criteria, such as determining which answer is the best for a problem. In a science problem-solving situation, creative thinking involves constructing a hypothesis and critical thinking involves testing that hypothesis.

Based on their structure, problems can be divided into two types: (1) well-defined or closed problems, and (2) ill-defined or open-ended problems (Aziza, 2019). Closed problems are problems that are presented in a "well-structured" form that is clearly formulated tasks. In this type of problem, there is one correct answer and it can be solved in a certain way (fixed ways). This type of problem includes problems with content-specific multiple-step routines and problems based on non-routine heuristics.

Multiple-step content-specific routine problems, also known as challenge problems, that are applied in learning for problem solving with the main goal of solving problems on certain topics. This type of challenge problems is used to assess a student's ability called higher order analytical thinking skills and non-routine closed problems can be used for thinking skills and heuristics and can also be used in teaching about problem solving (Yazar Soyadı, 2015). This learning emphasizes the use of heuristic ways such as guess and check, work systematically, solve simpler cases, tabulate data, and look for a pattern. An open problem or ill-defined problem is characterized by the presence of a problem statement, problem formulation, objective statement, and/or less clearly specified operations. Depending on the features of the situation, a problem may be either well defined or weakly defined. These open-ended problems are often short open-ended problems. Problems shaped like this can be used in learning mathematics through problem solving that emphasizes learning mathematical concepts and skills. Bahar & June Maker found that short open-ended problems can be used to quickly check students’ conceptual understanding and thinking. This open-ended situation allows students to show what they know and also allows teachers to gain more
information about how their students solve mathematical problems. Students use the measurement knowledge they have gained, they use various “division” ways and different models and schema notations to support their process of thinking (Bahar & June Maker, 2015).

The level of complexity of a mathematical problem is highly dependent on the sophistication of the knowledge and skill of students required to solve the problem. Florida Department of Education divides the complexity of mathematical problems into 3 categories, namely: (1) low complexity problems, (2) moderate complexity problems, and (3) high complexity problems (high complexity). The complexity of a mathematical problem is highly dependent on the knowledge and skills of students needed to respond or solve the problem (Kontorovich et al., 2012). Problems with low complexity emphasize the aspect of remembering the concepts and principles that have been learned by students. This problem is generally characterized by demands for students to carry out some mechanical procedures and do not bring up original methods or solutions from students. Artemenko et al. (2019) stated problems with this low level of complexity consist of solving one-step problems, such as computational operations; evaluating variable expressions, such as determining the value of a variable; recognizing or constructing equivalent representations; recalling or recognizing facts, terms, or properties; synthesizing information from graphs, tables, or figures; exploring appropriate units or tools for common measurements; or performing single-unit conversions.

Problems with moderate complexity require more flexible thinking rather than problems with low one. The Problems require a response beyond the habitual of students and usually have more than one step to arrive at a solution. Students are required to decide what should be done by employing skills and knowledge from various domains. Some examples of skill that can be employed to respond to the problems are solving a problem requiring multiple operations, solving a problem involving spatial visualization and/or reasoning, using graph and using different representations.

Problems with high complexity are very demanding of students’ thinking. Students must involve the use abstract reasoning, critical thinking and creative thinking. To respond the problems, students should have some abilities, such as solving a problem in more than one-way; describing different representations, generalizing an algebraic or geometric pattern, and explaining and justifying a solution to a problem, formulating a mathematical model for a complex situation, and producing a deductive argument. If it is related to the problem definition discussed in the previous section, it appears that the three types of "mathematical problems" according to the Florida Department of Education, not all of those are mathematical problems. Based on the definition of the problem, which includes mathematical problems are problems with medium and high complexity. While the problem with low complexity is not a mathematical problem, but an exercise (exercise).

Jonassen & Carr (2020) escribes the factors that support the level of difficulty of test items and mathematical proficiency displayed in PISA. These factors include: (1) interpretation, (2) reflection, (3) representation, (4) mathematical skills, and (5)
Mathematical argumentation needed to reach a solution. Interpretation and reflection are needed to complete a test item related to the context of the problem presented. Through interpretation and reflection, students can determine whether the context of the problem they are facing still needs to be constructed or whether the mathematical construction is already available. The type of representation activity required to complete a test item can be in the form of building a representation model or changing from one representation model to another. While the type and level of mathematical skills required to complete a test item can be in the form of simple mathematical skills or mathematical skills that involve more advanced mathematical knowledge, such as problem solving. The first type of mathematical skills usually deals with single-step problems, and the second type of mathematical skills usually deals with multi-step problems. Polya identified a number of different sorts of methods based on the techniques required to solve mathematical problems, including looking for patterns, considering a simpler version of the issue, creating a table, and formulating an equation.

Finally, the types and levels of problems based on the mathematical arguments needed to solve them are problems without the need for arguments; problems that apply arguments that students know well; and problems in which students have to construct their own mathematical arguments, or examine arguments given or prove. Polya identified two categories of problems; those are problems to discover and problems to prove. Stacey (2015) also identified two kinds of mathematical problems, namely mathematical problems in the real world and mathematics problems. Meanwhile, there is a study define problems into four types, namely closed-routine, closed non-routine, open-ended with known goals and open-ended investigations/projects (Fai, 2005). Furthermore, NCTM implicitly stated that mathematical problems can arise in mathematics context and outside mathematics context [2]. Vondracek & Pittman stated that there are three types of mathematical problems, namely procedural, conceptual, and application. Procedural problems require students to: (1) select and apply correct operations or procedures, (2) modify procedures, if necessary, (3) read and interpret graphs, charts, and tables, (4) estimate and sort numbers, and (5) use formulas (Ling et al., 2019).

Conceptual problems require students to: (1) understand basic mathematical concepts, (2) identify and apply mathematical concepts and principles, (3) compare and integrate concepts and principles, (4) interpret and apply symbols and mathematical terms, and demonstrate an understanding of the relationship between numbers, concepts, and principles.

CONCLUSION

Students’ problem-solving ability is very importance in learning mathematics. The students' mathematical problem-solving ability is influenced by three main variables, namely subject variables, task variables, and situation variables. In every mathematical problem-solving activity, the three main variables do not stand alone separately, but they are involved interactively and determine greatly the success or failure of students’ mathematical problem solving. Issues
related to the development of test instruments; it can be stated that there are many kinds of types of problem solving. Based on the orientation, the task can be divided by two (at least), namely problem for generating concept and problem for applying concept. Based on the structure, the task can be divided by two, namely ill-structure and well-structure. Based on the essential steps to solving, the task can be divided by one-step solution, two-step solution, and multi-step solution.

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