AN EXPLORATION STUDENT'S ERRORS IN SOLVING TRIGONOMETRIC RATIO PROBLEMS WITH ITS SCAFFOLDING

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Abstract
Trigonometry is an essential topic in learning mathematics, but in fact there are still identified student errors in solving trigonometry problems. Based on these problems, this study aims to describe students' errors in solving trigonometric problems, especially on the topic of trigonometric ratios and its scaffolding. This study uses a qualitative method with a case study approach. Collecting data through observation, tests and interviews. The subjects of this study were 35 students of class X in one of the senior high schools in Kutai Kartanegara Regency, East Kalimantan. The results of this study are that there are seven types of errors made by students in solving trigonometric ratio problems, namely: (1) error in understanding the concepts; (2) error in setting operation; (3) error in performing calculation; (4) error in applying the principle; (5) error in writing algorithm; (6) random response; and (7) error in drawing. Based on the results of these studies, it can be concluded that the forms of scaffolding that can be given are: explaining, reviewing, restructuring, and developing conceptual thinking.

Keywords: Error, scaffolding, trigonometric ratio

Abstrak
Trigonometri merupakan mata pelajaran yang esensial dalam pembelajaran matematika, namun pada fakta masih terdapat kesalahan-kesalahan siswa yang teridentifikasi dalam mengatasi masalah trigonometri. Berdasarkan masalah-masalah tersebut, penelitian ini bertujuan untuk mendeskripsikan kesalahan-kesalahan siswa dalam menyelesaikan masalah trigonometri khususnya pada topik rasio trigonometri beserta scaffoldingnya. Penelitian ini menggunakan metode kualitatif dengan pendekatan studi kasus. Pengumpulan data melalui observasi, tes dan wawancara. Subjek penelitian ini adalah 35 siswa kelas X di salah satu SMAN di Kabupaten Kutai Kartanegara, Kalimantan Timur. Hasil penelitian ini yakni terdapat tujuh jenis kesalahan yang dilakukan siswa dalam menyelesaikan masalah rasio trigonometri, yaitu: (1) kesalahan dalam memahami konsep; (2) kesalahan dalam menetapkan operasi; (3) kesalahan dalam melakukan perhitungan; (4) kesalahan dalam menerapkan prinsip; (5) kesalahan dalam menuliskan algoritma; (6) jawaban sembarangan; dan (7) kesalahan dalam menggambar. Berdasarkan hasil penelitian tersebut, dapat disimpulkan bahwa bentuk scaffolding yang dapat diberikan yaitu explaining, reviewing, restructuring, dan developing conceptual thinking.

Kata kunci: Kesalahan, rasio trigonometri, scaffolding

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INTRODUCTION
Trigonometry is one of the essential topics in mathematics education. This is supported by the statement that trigonometry is an essential topic and cannot be separated from mathematics topics at the high school level (Huda & Qohar, 2021; Jelatu, Kurniawan, Kurnila, Mandur, & Jundu, 2019; Kamber & Takaci, 2018; Yang & Sianturi, 2017). The topic of trigonometry also has an important role outside the field of mathematics education, namely contributing to the development of science and technology and in everyday life such as measuring the height of buildings or buildings, flight engineering, helping to create a robot, and playing a role in the health sector (Emmanouil, Wei, & Dai, 2016; Goel, 2015; Jelatu et al., 2019; Mulyawati, Salmawati, Subianto, & Wafdan, 2017; Yang & Sianturi, 2017). In addition, the trigonometry topic has an important role because it is the topic that is tested as the final assessment in learning mathematics (Brijlall & Niranjan, 2015; Law, Shahrill, & Mundia, 2015). Therefore, trigonometry is one of the important components of mathematics learning and becomes the topic to be tested as a final assessment in mathematics learning.

Although trigonometry has an important role and becomes the topic that is tested as a final assessment. However, in reality trigonometry is often assumed to be a very difficult subject (Kamber & Takaci, 2018; Walsh, Fitzmaurice, & O’Donoghue, 2017) so that the errors experienced by students in the process of learning mathematics, especially trigonometry topic are identified. As was discovered by Rohimah & Prabawanto (2019), that students make errors because they have difficulty in understanding angles related to trigonometry, and in calculating or operating trigonometric problems. Other findings by Usman & Hussaini (2017), namely errors made by students include understanding, transformation errors, and process skill errors. The cause of students making errors comes from the lack of student mastery of the prerequisite topic (Brijlall & Niranjan, 2015). In addition, another cause of students making errors comes from the learning resources that students receive (Chigonga, 2016; Rohimah & Prabawanto, 2019). Therefore, errors made by students are caused by several factors, namely factors that come from within students, namely due to incomplete understanding of students' concepts, and factors that come from outside, namely the learning resources that students receive.

Hamzah et al (2021) has conducted a systematic literature review (SLR) related to the topic of research on student misconceptions and errors in learning trigonometry in the last 10 years. The findings show that research that only aims to identify student errors and misconceptions dominates more than research that aims to identify and provide solutions to minimize errors made by students. Therefore, there is a need for a study that does not only identify student errors, but is able to provide solutions to minimize them. One solution that can be given to minimize student errors is to provide sufficient assistance to students.

The provision of assistance in question is based on the theoretical concept of Vygotsky, a psychologist from Russia who is known as an expert who discusses constructivism theories. One of the ideas from Vygotsky’s theory referred to in this research is scaffolding. Scaffolding is assistance provided by the teacher to students in
solving a problem that aims to help students through their Zone of Proximal Development in order to achieve learning objectives (Vygotsky, 1978). On the other hand, Doo et al (2020) visualizes scaffolding as teaching children how to learn to ride a bicycle, namely when children start pedaling, parents need to hold the chair firmly so that the child does not lose control. Furthermore, when the child begins to learn to balance the bicycle, the parents need to let go of the child and let him ride the bike without any further assistance. Therefore, the final result to be achieved in scaffolding is to guide students to be able to achieve mathematical competence and complete tasks or problems independently (Bakker, Smit, & Wegerif, 2015; Hartman, 2002; van de Pol, Volman, Oort, & Beishuizen, 2015). Scaffolding is one solution in improving understanding of concepts, helping students' thinking processes in solving problems, and constructing new knowledge (Lin et al., 2012; Maksić & Jošić, 2021; Paruntu, Sukestiyarno, Priyono, & Prasetyo, 2018; Rakhim, Kartono, & Supriyadi, 2022; Susilowati, Rochmad, & Rusilowati, 2021). According to Anghileri (2006) in the learning process scaffolding is divided into four components, namely explaining, reviewing, restructuring, and developing conceptual thinking. In this study, referring to the four components that were initiated by Anghileri. According to Khusna (2021), explaining is an activity to convey concepts that are not mastered by involving a small amount of student contribution, reviewing is an activity to refocus on important information on a task or problem, restructuring is an activity to bring up a modification of ideas on a given task or problem, and finally developing conceptual thinking is an activity involving students in reaching a broader conceptual in students' thinking.

Based on the results of a literature review study conducted by Hamzah et al (2021) that for the topic of research on student misconceptions and errors in learning trigonometry in the last 10 years, it was found that research aimed at identifying student errors dominates compared to research that provides solutions to minimize errors made by students. Then, for the types of errors used are errors based on Newman’s error, Reading, Comprehension, Transformation, Process Skill, Encoding, Language, Careless, Error Type I (understanding the meaning of the question), Error Type II (concept), and Error Type III (calculation).

Thus, this study aims to explore the errors made by students in solving trigonometric problems and the scaffolding that will minimize them.

**RESEARCH METHODS**

This research is a type of qualitative research with a case study approach. The subjects of this study were 35 students of class X in one of the senior high schools in Kutai Kartanegara Regency, East Kalimantan. Subject selection is based on suggestions from the teacher, namely the ability of students who are different and can meet the abilities of the low, medium and high categories. The stages in this research are divided into three, they are the preparation stage, the implementation stage, and the final stage. First, the preparation stage which consists of the process of selecting research subjects, licensing, and preparing research instruments. Second, the implementation stage which consists of the process of collecting research
data, namely the results of observations, diagnostic tests and interviews. Last, the final stage is the process of drawing conclusions on the results of the analysis in the previous stage.

The main instrument in this study was the researcher himself while the supporting instruments were observation sheets, diagnostic tests and interview guidelines. The observation sheet is used to record the information obtained during the learning process in the classroom. The diagnostic test in the form of a description question consists of 5 items that are used to identify student errors in solving trigonometric ratio problems. While the interview guide is in the form of a list of open-ended questions so as to provide comprehensive information.

The data collection technique used in this research is through a triangulation process, which is a combination of observation data, documentation studies, and interviews. The researcher made observations to find out how the learning process in the classroom was, and to identify errors that occurred so that these errors could be compared with the errors found in the student’s work. Then the subject was given a description question consisting of 5 items regarding the problem of trigonometric ratios with a duration of 40 minutes. The results of student work as written documentation were clarified through the interview process, researchers conducted interviews with several selected students. With the criteria of the student solving the problem on the written test given, the student is dominant in making errors on the given problem, and based on the advice of the mathematics teacher concerned. The purpose of the interview process is to clarify the results of student work and to identify forms of scaffolding that can be provided. The form of scaffolding used in this study is based on what was stated by Anghileri (2006) namely explaining, reviewing, restructuring, and developing conceptual thinking.

The data analysis technique used in this study was carried out through the process of data reduction, data display, and drawing conclusions

a. Data Reduction
Data reduction carried out in this study is to remove information that is not needed in the study, namely data from observations, diagnostic tests, and interview results. Data reduction aims to identify and classify student errors in solving trigonometric ratio problems. In this case, data reduction is viewed from the types of errors that have been determined by the researcher. The types of errors used in this study are: 1) Errors in understanding the concept; 2) error in setting operation; 3) errors in performing calculations; (4) errors in applying principles; 5) errors in writing algorithms; and 6) random response.

b. Data Display
The presentation of data in this study is data on student work on diagnostic tests. The identified errors are marked in red and blue lines, then an explanation is given to these errors and accompanied by the form of scaffolding that can be given to minimize them.

c. Conclusion Drawing
Based on the previous stage, the data was analyzed to obtain conclusions in the form of what errors were made by students in solving trigonometric ratio problems and how scaffolding could be given.
RESULTS AND DISCUSSION

The research was carried out in 4 meetings with details of three meetings for learning activities and one last meeting for testing trigonometric ratio problems. The errors made by students in solving trigonometric ratio problems found by researchers during research and the form of scaffolding given are as follows.

### Error in Understanding Concept

The error in understanding the concept found by the researcher was that the students misunderstood the definition from the point of depression and incorrectly wrote the definition of the trigonometric function. The question of diagnostic tests and the student’s answer can be seen in Table 1.

#### Table 1. The first question and student’s answer

<table>
<thead>
<tr>
<th>Question</th>
<th>Student’s Answer</th>
</tr>
</thead>
<tbody>
<tr>
<td>Let ( \tan x = 1 ) (( x^\circ ) is an acute angle), find the value of ( \csc x, \sec x ) and ( \cot x )!</td>
<td>![Image]</td>
</tr>
</tbody>
</table>

Based on student’s answer (marked in red), it shows that inaccuracy of students in writing the concept of trigonometric ratios, namely writing sides in the meaning of "oblique, front, and side". For error in understanding concepts, the form of scaffolding that can be given is reviewing. At the reviewing stage, students are asked to reveal what is meant by the work they have written so that students can find out where their errors are. For error in understanding concepts, the form of scaffolding that can be given is reviewing. At the reviewing stage, students are asked to reveal what is meant by the work they have written so that students can find out where their errors are. For error in understanding concepts, the form of scaffolding that can be given is reviewing. At the reviewing stage, students are asked to reveal what is meant by the work they have written so that students can find out where their errors are.

### Error in Setting Operation

The error in setting operation found by the researcher was that the student was wrong in using or setting the operation. The question of diagnostic tests and the student’s answer can be seen in Table 2.

#### Table 2. The second question and student’s answer

<table>
<thead>
<tr>
<th>Question</th>
<th>Student’s Answer</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nur with a height of 100 cm is observing the top of a building with a building height of 25 m. In the initial position, Nur observed the top of the building with an elevation angle of 30(^\circ), then Nur walked as far as ( x ) meters towards the building and looked back at the top of the building with an elevation angle of 60(^\circ). Determine the distance Nur to the building from Nur's initial position!</td>
<td>![Image]</td>
</tr>
</tbody>
</table>
Based on student’s answer (marked in red), it shows that the student's process in determining the value is by dividing each of the two sides by 3, but the final result of the student is $75\sqrt{3}$. For error in setting operation, the form of scaffolding that can be given is explaining, reviewing and restructuring. At the explaining stage students are asked to re-read the information from the problem given carefully and thoroughly, so that students can understand what was asked and what is known from the problem given. After students have understood what is known and asked from the questions, the next step is reviewing. The teacher asks students to re-check the answer whether it is in accordance with what is desired from the question, then asks students to make an illustrative image of the problem. The last stage is restructuring, where the teacher helps students with the understanding they already have in determining the value of a variable so that students can find solutions to the problems given.

Error in Performing Calculation

Errors in performing calculations found by the researchers were that students were wrong in calculating the result of an operation and students were wrong in calculating the value of trigonometric functions for special angles. The question of diagnostic tests and the student’s answer can be seen in Table 3.

Table 3. The third question and student’s answer

<table>
<thead>
<tr>
<th>Question</th>
<th>Student’s Answer</th>
</tr>
</thead>
</table>
| Look at the following picture! | ![Picture](image)
| If $BC = CD$ and $\tan x = \frac{3}{2}$, find the value of $\sin \angle B$, $\cos \angle B$ and $\tan \angle B$! | ![Answer](image) |

Based on student’s answer (marked in red), it shows that the student completing the operation $\sqrt{9 + 4}$ is $\sqrt{13}$. For error in performing calculations, the forms of scaffolding that can be given are reviewing and restructuring. In the reviewing stage, the teacher asks students to correct or review the results of their work, whether it is in accordance with what is desired from the problem and correcting the calculation process that is less precise. Then the teacher gives instructions to students in order to understand the key to the problem given. The last stage is restructuring, where the teacher gives direction to students so they can find solutions to the problems given.

Error in Applying the Principle

The errors in writing or applying the principles found by the researcher were that the students were wrong in writing the Pythagorean theorem and were wrong in the steps to rationalize the root form of fractions. The question of diagnostic tests and the student’s answer can be seen in Table 4.
Based on student’s answer (marked in red), it shows that the students’ inaccuracy in writing the expression for the radical form of the Pythagorean theorem becomes \( b = a^2 + c^2 \). For errors in applying these principles, the forms of scaffolding that can be given are reviewing and developing conceptual thinking. At the reviewing stage, the teacher asks students to correct or review the results of their work in order to find errors that have been made. Then the last stage is developing conceptual thinking, where the teacher asks students to recall the prerequisite topic that has been studied previously. In this case, the prerequisite topic in question is the Pythagorean theorem topic.

**Error in Writing Algorithm (Defective Algorithm)**

The errors found by the researchers were that the students did not write down the completion steps systematically, the students wrote the wrong initial steps for completion and the students incorrectly simplified the final results obtained. The another student’s answer who made error in writing algorithms for second question can be seen in Figure 1.

![Figure 1](image)

**Figure 1.** The student’s answer made error in writing algorithms

Based on Figure 1 for step number (1) shows that the initial step of completion of students who intend to write the tangent function on a right triangle \( \Delta ABC_1 \), but students only write \( \Delta ABC_1 = \frac{AB}{BC_1} \) and the student’s completion step is not complete.

For error in writing the algorithm, the forms of scaffolding that can be given are explaining, reviewing and restructuring. At the explaining stage students are asked to re-read the information from the given problem and ask students to state what they know...
from the problem. After students realize the location of the error from the information used, the next stage is reviewing. The teacher asks students to re-check their answers so that students realize where their errors. The last stage is restructuring, where the teacher asks students to reconstruct the algorithm written by the student so that it does not deviate from the actual mathematical concept.

**Random Response**

The random answer errors found by the researchers were that students wrote answers carelessly, that is, the answers written were unclear and the answers were not related to the problem given. The another student’s answer who make errors in drawing for third question can be seen in Figure 2.

![Figure 2. Answers of students who make errors in drawing](image)

Based on Figure 2, it shows that the student wrote that \( \tan \frac{\theta}{2} = 5 \text{ rad} \). This is not relate with the question. For the random response, the forms of scaffolding that can be given are explaining, reviewing and restructuring. At the explaining stage students are asked to re-read the information from the given problem carefully and thoroughly, so that students can understand what is being asked and what is known from the problem given. After students have understood what is known and asked from the questions, the next step is reviewing. The teacher asks students to re-check the answer whether it is in accordance with what is desired from the question, then asks students to make illustrative pictures of the problem and the teacher can emphasize important information. The last stage is restructuring, where the teacher can simplify an existing problem, in order to build student understanding.

**Error in Drawing**

The random answer errors found by the researcher were students writing answers carelessly, namely the answers written were not clear and the answers were not related to the problem given. The another student’s answer who make error in random response second question can be seen in Figure 3.

![Figure 3. Answers of students who make error in random response](image)

Based on Figure 3, it shows that the length of 100 cm is shorter than 30 cm long. In addition, the 60° angle drawn by the students shows that the 60° angle is an obtuse angle and the right angle symbol made by the students is not clear. nts' errors in the solving algorithm.

In the drawing error, the forms of scaffolding that can be given are explaining, reviewing and restructuring. At the explaining stage students are asked to re-read the information from the problem given carefully and thoroughly, so that students can understand what was asked and what is known from the problem given. After
students have understood what is known and asked from the questions, the next step is reviewing. The teacher asks students to re-check the answer whether it is in accordance with what is desired from the question, then asks students to make illustrative pictures of the problem and the teacher can emphasize important information. The last stage is restructuring, where the teacher can simplify an existing problem, in order to build student understanding.

The research findings indicate that there are other types of errors outside of the six types of errors that have been determined, namely student errors in representing problems into mathematical models (pictures). For this reason, the error is labeled an error in drawing. Some students have difficulty in representing questions in the form of pictures so that strategies can be determined to be used in solving problems. This is in line with research by Dewanto et al (2017) which found that students wrote the concept of trigonometric ratios in the meaning of "front and side". In addition, research by Dewanto et al (2017) also found that students wrote down the concept of trigonometric ratios in "de, sa, and mi".

b. Error in setting Operation
The cause of students making errors in setting operations is because students do not understand how to determine the value of a variable on the topic of one-variable linear equations. This is what causes students to be wrong in determining the operation to determine the final result, even though students understand the strategy in solving problems. This is in line with the findings of Arhin & Hokor (2021) that students have difficulty in determining the value of a variable or in other words students have difficulty in solving trigonometric problems in the form of equations. The findings explain that the topic of algebra as a basic concept that must be mastered by students, and the learning process should be adapted to the times that are not just transferring knowledge or traditional approach models.

c. Error in Performing Calculations
The cause of students making errors in doing calculations is that students are in a hurry and are not careful in solving problems. This is in line with findings of Setiawan (2021) that
students make errors because they are not careful in answering questions.

d. Errors in Applying the Principles
The cause of students making errors in applying the principles is the limited understanding of students to the formulas being studied. Students only focus on “if finding the hypotenuse is to add up the squares of the sides of the triangle”. This happens because of the explanation of the formula without any construction process. So that students just memorize it without understanding the concept of the formula. This is in line with the findings of Rachman & Purwasih (2021) that students understand the principles but students make errors in writing these principles, namely the students' inaccuracy in writing the expression of the square root of the trigonometric function.

e. Error in Writing Algorithm
The cause of students making errors in writing algorithms is that students are afraid of not having time to solve the problem so that students are not careful in writing answers. This finding is in line with that found by Setiawan (2021) that students make errors in applying trigonometric comparisons to right triangles because students do not re-check the answers that have been obtained.

f. Random Response
The cause of students making errors is students do not understand the topic and students only remember the topic about changing angles to radians. This is in line with the results of research by Kamber & Takaci (2018) which states that the majority of students are easier to remember and apply when the angle is in radians.

g. Error in Drawing
The cause of students making errors in drawing is because students have difficulty in interpreting the problem into a mathematical model, and students do not understand the concept from an elevation point of view.

CONCLUSION AND SUGGESTION
Based on the results and discussion, it can be concluded that:
The errors made by Grade 10 students in one of the senior high schools in in a district located in the Province of East Kalimantan are errors in understanding concepts, errors in determining operations, errors in calculating, errors in applying principles, errors in write down algorithms, drawing errors and random answers.

The forms of scaffolding given to each subject are not the same, namely explaining, reviewing, restructuring, and developing conceptual thinking. It should be noted that the form of scaffolding provided is only a recommendation and has not been applied to research subjects. Therefore, it is hoped that future researchers can consider these recommendations to be applied to students and better be able to develop new learning designs for inculcating the concept of trigonometric ratios to students.

In addition, the findings in this study are able to become the basis for further research, namely the critical paradigm, which aims to make changes to existing learning designs or didactic designs. So that students' errors or misconceptions about the concept of trigonometric ratios are minimized.

REFERENCES


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