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BRIDGING ARITHMETIC TO ALGEBRA: IMPLEMENTING AN E-MODUL THROUGH DIDACTICAL DESIGN RESEARCH

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Abstract

Students' difficulties in understanding algebraic concepts often stem from a procedural approach to arithmetic without deep conceptual grounding. This study aims to develop and implement an interactive e-module titled *"Jembatan Aritmatika"*, designed to help students build a conceptual transition from arithmetic to algebra by deepening their understanding of the equal sign (=) as a relational symbol. The study adopts a *Didactical Design Research (DDR)* approach, consisting of three main phases: *preliminary design*, *experimental teaching*, and *retrospective analysis*. The participants were upper-grade elementary school students who engaged with the e-module across several learning sessions. Instruments included pre-test and post-test questions, observation sheets, and student response questionnaires. The results showed that the e-module effectively enhanced students' understanding of the relational meaning of the equal sign and introduced them to the fundamental structure of algebraic equations in a gradual and meaningful way. Strategies such as visualization, contextual problem-solving, and the progressive use of symbolic representation proved helpful in supporting students' reasoning about quantitative relationships. The study concludes that the *"Jembatan Aritmatika"* e-module holds promise as an effective learning medium for facilitating students' conceptual transition from arithmetic to algebra.

Keywords: Algebra; arithmetic; didactical design research; e-module; equal sign.

Abstrak

Kesulitan siswa dalam memahami konsep aljabar sering kali berakar pada pemahaman aritmatika yang bersifat prosedural, tanpa makna konseptual yang kuat. Penelitian ini bertujuan untuk mengembangkan dan mengimplementasikan sebuah e-modul interaktif bertajuk "Jembatan Aritmatika" *yang dirancang untuk membantu siswa membangun jembatan konseptual dari aritmatika ke aljabar melalui perluasan makna tanda sama dengan (=). Penelitian ini menggunakan pendekatan Didactical Design Research* (DDR) *yang terdiri atas tiga tahap utama: preliminary design, experimental teaching, dan retrospective analysis. Subjek penelitian adalah siswa sekolah dasar kelas atas yang mengikuti pembelajaran menggunakan e-modul dalam beberapa sesi. Instrumen yang digunakan mencakup soal pre-test dan post-test, lembar observasi, serta kuisioner respon siswa. Hasil penelitian menunjukkan bahwa e-modul ini mampu meningkatkan pemahaman siswa terhadap makna relasional tanda sama dengan dan mengenalkan mereka pada struktur dasar persamaan aljabar secara bertahap dan bermakna. Strategi visualisasi, konteks kontekstual, serta penggunaan simbol secara progresif terbukti membantu siswa merepresentasikan hubungan antar kuantitas. Simpulan dari penelitian ini menunjukkan bahwa e-modul* Jembatan Aritmatika *berpotensi menjadi media pembelajaran yang efektif dalam mendukung transisi konseptual dari aritmatika menuju aljabar.*

***Kata kunci**: Aljabar; aritmatika; e-modul; penelitian desain didaktis; tanda sama dengan.*

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INTRODUCTION

The transition from arithmetic to algebra is one of the conceptual challenges faced by students in elementary and secondary education (Ardiansari et al., 2022). Many students struggle to understand algebraic symbols because they lack a strong conceptual foundation for the meaning of arithmetic operations and relations (Ardiansari et al., 2023). This hinders their ability to think abstractly and construct equations to solve more complex mathematical problems.

Previous research has shown that difficulties in learning algebra often stem from a limited understanding of the equals sign (=) as a relation, not simply a signal for calculating results (Ardiansari, 2023; Carpenter, Franke, & Levi, 2003; Wardat et al., 2021). Students generally view the equals sign as a command to find the final result, rather than as a statement that two expressions have the same value. However, a relational understanding of the equals sign is crucial as a foundation for algebraic thinking (Blanton et al., 2015; Sibgatullin et al., 2022). Furthermore, the use of learning media that emphasizes exploratory and problem-based activities has been shown to help strengthen conceptual understanding ((Iriani et al., 2025); (Nabila, Aziz, & Suprapto, 2025)). However, studies of basic mathematics learning media in Indonesia still focus largely on procedural skills and have not systematically built a conceptual bridge from arithmetic to algebra. Not many didactic design research-based learning tools have been developed that explicitly design students' thinking stages from comparing, exchanging, combining information, to constructing equations. This is the gap that this study attempts to fill.

The problem raised in this study is the unavailability of interactive and contextual teaching materials that can guide elementary school students to understand algebraic concepts gradually through reinterpreting the equal sign (=). The lack of learning media that bridges understanding from concrete representations to symbolic forms causes students to have difficulty building algebraic thinking structures. As a solution, an interactive e-module entitled "Jembatan Aritmatika" was developed, designed with a didactic design research approach. This module presents step-by-step activities starting from price comparisons, exchange of goods, exploration of combinations, to notation and equations, all of which aim to build the relational meaning of the equal sign. This e-module also uses everyday contexts that are close to the students' world, so it is expected to increase their engagement and understanding.

The purpose of this study is to implement and evaluate the effectiveness of the "Jembatan Aritmatika" e-module in building students' understanding of basic algebraic concepts, particularly through the meaning of the equal sign (=). This study is expected to contribute to the development of basic mathematics teaching materials based on didactic design and strengthen the foundation of algebraic thinking from an early age.

METHODS

This research uses the Didactic Design Research (DDR) approach developed by Suryadi (2019). This approach aims to develop and examine didactic designs that are appropriate to student characteristics and the context of mathematics learning. There are three systematic stages in DDR (Suryadi, 2023).

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1. Didactic Situation Analysis (DSA)

This stage begins with a conceptual analysis of students' difficulties in understanding basic algebraic concepts, particularly the meaning of the equal sign (=). Researchers conducted a literature review, curriculum review, and classroom observation. Based on these findings, the initial design of the "Jembatan Aritmatika" e-module was developed, which includes step-by-step activities from arithmetic to algebra.

Figure 1. Overview of the E-Module "Jembatan Aritmatika" in Indonesian

2. Metapedadidactic Analysis (AM)

At this stage, the e-module was implemented through classroom learning in two stages:

a) Pilot experiment, where the initial design was tested on a small group (5 students) to assess initial responses and the feasibility of the activity.
b) Teaching experiment, where the e-module was implemented on a full class scale (24 students). The researcher observed the learning process and flexibly adjusted the learning strategy based on student responses to each activity.

3. Retrospective Analysis (AR)

After implementation is complete, researchers reconstruct the learning process based on all collected data (test results, observations, videos, interviews). The primary focus is on observing the development of student understanding and assessing the effectiveness of the e-module design. The results of this analysis are used to revise or strengthen the learning design.

This research was conducted at an elementary school in Probolinggo. Twenty-four fifth-grade students, aged 11–13, participated in the study. Subjects were selected purposively, considering that students had studied basic arithmetic but had not been formally introduced to algebraic concepts. The school was also selected because it had adequate digital infrastructure to integrate e-modules into its learning.

Several supporting instruments used in this study include: a) The “Jembatan Aritmatika” E-Module, as a developed activity tool; b) Pretest and posttest questions, designed to measure the development of students’ understanding of the equal sign and early algebraic concepts (can be seen in Table 1 and 2); c) Observation sheets and field notes, to record students’ responses and the dynamics of the learning process; d) Video recordings of learning, used to document students’ interactions and strategies in detail; and e) Semi-structured interviews, conducted with several students to explore their

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conceptual understanding in depth. Instrument validation was carried out through expert judgment from two mathematics lecturers, one linguist, one learning media expert, and one experienced elementary school teacher. Improvements to the questions and activities were made based on suggestions from the experts.

Table 1. Pre-test questions

No.	Questions	Competencies Measured	Description
1	Complete: 8 = ___ + 5	Understanding the equivalent relationship of the equal sign (=)	Uncovering early misconceptions
2	Which one is correct? a) 10 = 7 + 3 b) 10 = 3 + 7 c) 10 = 10 d) All correct	Understanding the relational meaning of the equals sign (not just the result)	Exploratory multiple choice
3	At the Honesty Shop, a pencil costs Rp. 150 and an eraser costs Rp. 250. What is the total price for 2 pencils and 3 erasers?	Adding quantities with contextual price units	Realistic story context
4	If we know: apple + sandwich = Rp34,000 Milo + sandwich = Rp42,000 apple + Milo = Rp28,000 What can you learn from this information?	Interpreting quantitative information systems	Assessing readiness for the system of equations
5	What do you think the “=” sign means in mathematics? Explain with an example.	Exploring students' views on the meaning of the equal sign	Exploratory open questions

Table 2. Post-test Questions

No.	Questions	Competencies Measured	Description
1	Complete: 8 = ___ + 5	Evaluation of understanding of the equal sign relationship	Reflective questions from the pretest
2	Which one is correct? a) 10 = 7 + 3 c) 10 = 10 b) 10 = 3 + 7 d) All correct	Evaluation of understanding of the relational meaning of the equal sign	Reflective questions from the pretest
3	In the Honesty Shop, the price of a pencil is Rp150 and an eraser is Rp250. If someone buys 2 pencils and 3 erasers, what is the total? Write it in the form of an equation.	Constructing mathematical models from story context	Symbolic representation
4	Rafi bought 2 t-shirts and 1 jacket for Rp30,000. If he exchanged 1 t-shirt for 1 jacket and had to pay an additional Rp6,000, what would be the difference in price between the t-shirt and the jacket?	Using substitution and forming a two-variable system	Logical context of the e-module
5	If: apple + sandwich = Rp34,000 Milo + sandwich = Rp42,000 apple + Milo = Rp28,000 Determine the price of each.	Solving a system of equations in three variables	Synthesis of contextual information

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No.	Questions	Competencies Measured	Description
6	The weight of the chickens is: Large + Medium = 10.6 kg Small + Large = 8.5 kg Small + Medium = 6.1 kg What is the total weight of the three chickens?	Compile and solve systems of equations	Notational representation problem

Data analysis techniques were adapted to the type of data collected, namely: pre-post tests were analyzed quantitatively using gain scores and distribution analysis based on student understanding indicators; observation data and field notes were analyzed qualitatively descriptively using coding to identify problem-solving strategies; while student interviews were analyzed using a thematic analysis approach to explore changes in meaning towards mathematical symbols and algebraic concepts. Data triangulation was carried out by combining findings from observations, videos, student work, and interviews to ensure the validity of the interpretation results.

RESULTS AND DISCUSSION

The results of this study are presented based on three main stages in the Didactical Design Research (DDR) approach, namely Didactic Situation Analysis (ASD), Metapedadidactic Analysis (AM), and Retrospective Analysis (AR).

4.1 Didactic Situation Analysis (DSA)

Preliminary analysis showed that the majority of fifth-grade students, 16 out of 24, had misconceptions about the equals sign (=). Many students interpreted the equals sign as "where the answer is" (operational view), rather than as a statement that both sides have equal value (relational view).

This is evident from several student answers, particularly in pretest questions 1, 2, and 5, which were explicitly designed to reveal students' initial understanding of the meaning of the equal sign. In the question "Complete: 8 = ___ + 5", 16 students filled in 8 in the blank, making it "8 = 8 + 5", and 4 students answered 13 because they considered the question to be like "8 + 5 = ___". This shows that students do not yet understand the equality relation. They tend to read the equal sign as a "command to calculate" rather than as a statement that the value of both sides must be the same. In question 2, namely "Which is correct? a) 10 = 7 + 3 b) 10 = 3 + 7 c) 10 = 10 d) All are correct", all students chose only (a) and no students chose "all are correct". This shows that students are not yet flexible in seeing the equivalent form of a number (for example: 10 can be written as 7 + 3 or 3 + 7), because they are still bound by the one-way form of the operation. Meanwhile, for question number 5: "What do you think the = sign means in mathematics? Explain with an example." Many students wrote answers like: "The equals sign means the result," or "the placeholder," without explaining that both sides must be equal. All of these answers demonstrate an operational view of the equals sign, not a conceptual understanding. This reinforces the assumption that this misconception is systematic and needs to be addressed didactically.

In addition, observations of arithmetic learning in the classroom showed that the approach used still focused on calculation procedures and

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had not directed students to the conceptual meaning of mathematical symbols. These result became an important basis in the Didactic Situation Analysis (ASD) stage of the Didactic Design Research (DDR) method, so that the e-module "Jembatan Aritmatika" wasdesigned to expand the meaning of the equal sign and build relational understanding through context, visualization, and simple equations.

The findings regarding students' misconceptions about the equal sign reinforce findings in previous literature that many elementary school students tend to interpret the equal sign operationally (operational view), namely as a "command to calculate" or "where the answer is located" (Faujiah et al., 2024; Fyfe, Matthews, Amsel, McEldoon, & McNeil, 2018; Stephens et al., 2021). This view causes students to focus only on the process of producing an answer and not on the relational meaning that two expressions have the same value (Kieran et al., 2016; McAuliffe et al., 2020). Research by Jupri (2016) also found that less than 40% of middle-grade students were able to recognize the relational meaning of the equal sign, while the majority of students considered the equal sign as a symbol that only marks the end of a calculation. This finding is in line with the pretest results in this study, where the majority of students were unable to solve problems that required a correct relational understanding, as well as express the meaning of the equal sign with procedural narratives.

In addition, observations of learning practices show that teachers mostly use “linear procedural” questions (e.g., 4 + 5 = ___), without providing variations in form (such as ___ + 5 = 9 or 9 = ___ + 5) that can help students build relational meaning. This indicates that students’ misconceptions are not only rooted in cognitive limitations, but are also influenced by teaching practices that do not provide space for exploration of mathematical symbols.

In response to these findings, the researchers developed an e-module “Jembatan Aritmatika” that includes a series of context-based activities such as exchanging goods, purchasing combinations, and strategies for constructing equations from stories. The module was designed with attention to the importance of didactic situations (Brousseau, 2002) in actively building students' conceptual meaning to build a relational understanding of the equal sign and as an initial bridge to understanding algebraic concepts.

4.2 Metapedadidactic Analysis (AM)

4.2.1 Pilot Experiment

Four fifth-grade students participated in the pilot experiment. Results showed that the contextual activities in the e-module successfully captured interest and sparked discussion. Students began to construct notation from the story and reason, such as "two glasses equal one jacket," although not all students were able to construct mathematical equations correctly. Based on this feedback, several adjustments were made to the e-module's presentation and problem narratives to simplify them yet remain challenging.

4.2.2 Teaching Experiment

After revisions, the e-module was implemented with 24 students in a regular class. Data collection was conducted through observations, video recordings, student work, and pre-post tests. Students showed improvement in constructing mathematical notation and solving equation-based problems. For

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example, students began using letter symbols to represent objects and realized that the equal sign indicates the equality of two quantities in different forms. Some students still experienced difficulty transitioning from the story context to symbolic forms. However, through class discussions and teacher scaffolding, their understanding gradually developed.

4.3 Retrospective Analysis (AR)

Pretest data showed that only 11% of students were able to correctly solve problems related to the meaning of the equal sign. After implementing the e-module, posttest scores improved significantly, with 79% of students demonstrating a relational understanding of the equal sign and being able to construct and solve simple equations.

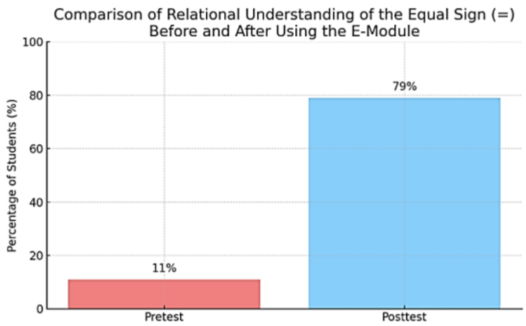

Figure 2. Comparison of Relational Understanding of the Equal Sign (=) Before and After Using the E-Module

The results of the study showed a significant increase in students' understanding of the meaning of the equal sign after participating in the learning using the "Jembatan Aritmatika" e-module. This improvement is evident in the pretest and posttest results (Figure 2), which show a shift from an operational view to a relational view. Initially, the majority of students viewed the "=" sign as a placeholder for the answer (e.g., 8 + 5 = ___), rather than as a statement that two sides have equal value. After using the e-module, students began to be able to solve context-based problems, construct equations from stories, and explain that the equal sign indicates equality of value, not simply the result of a calculation. For example, in the problem "8 = ___ + 5," students answered correctly and were able to explain that both sides must have equal value, namely 8 = 3 + 5.

This improvement demonstrates the effectiveness of the contextual and visual approach-based e-module in addressing misconceptions and building conceptual understanding of the equal sign as a foundation for the transition to algebra. This is in line with findings Chesney et al. (2018), Hornburg et al. (2015) and McNeil et al. (2017) which show that elementary school students often interpret the equal sign as “where the answer is” rather than as a sign of a relationship, so systematic intervention is needed to change this perspective.

The contextual approach used in this e-module provides anchored meaning for students, so that mathematical symbols do not stand alone, but are connected to real situations. This is reinforced by researchers who emphasize the importance of linking mathematical symbols to meaningful meanings through concrete contexts and visual representations ((Harbour et al., 2016); (Hitt et al., 2016); (Hornburg et al., 2021); (Kızıltoprak & Köse, 2017). Furthermore, activities in the e-module that include exchanging goods, weighing chickens, and price combinations in everyday stories provide space for students to experience the process of generalization and formalization of concepts, which is the core of the transition from arithmetic to algebra (Powell, 2012). Thus, the use of

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contextual and progressive e-modules can significantly help students develop strong conceptual and representational understanding, while also providing a foundation for mastery of algebraic concepts at a more advanced level.

Video and interview analysis revealed a shift in students' thinking from direct arithmetic to relationship analysis strategies. For example, when given a story about price combinations, students were able to construct a system of equations and solve them using a simple substitution method. In the teaching experiment session (Meeting 2), a moment was recorded when a student initially tried to add the prices directly without relational thinking. However, after being prompted by the teacher's question, such as "*What is the relationship between apples and Milo based on the information provided?*", the student began constructing mathematical sentences such as: "*If apples and sandwiches cost thirty-four thousand... Then Milo and sandwiches cost forty-two thousand... So we can subtract them to get the difference.*" Students then write answer like in Figure 3.

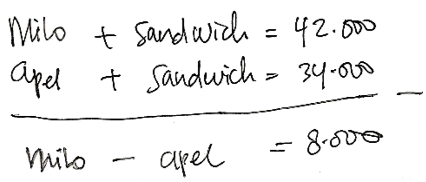

Figure 3. Example of student answers

Some students were able to realize that by subtracting the two equations, they could eliminate the common variable (Sandwich) and discover the relationship between apples and milo. This strategy reflects an understanding of early algebraic structures and relational thinking skills, not just procedural execution. According to Mirin (2019), a relational understanding of mathematical operations, including the use of the equal sign to express equality, is an important foundation for developing algebraic concepts. Similarly, Powell (2012) showed that students who develop a relational understanding are better equipped to understand and form mathematical equations and are more flexible in their solution strategies. This change also supports the findings of Kindrat and Osana (2018) who stated that the transition from arithmetic to algebra is not just about symbols, but how students see the relationships between values and information. In other words, the use of strategies such as parallel information subtraction indicates the development of a more conceptual algebraic thinking structure.

During a post-lesson interview, one student (initials: S1) said:
"*Before, when I had a problem like that, I was confused and just wanted to add more. But now I understand. I can compare things like: Milo is more expensive than apples, and then just find the difference.*"

This statement also reflects a shift in thinking from procedural to analytical, as well as an initial understanding of the use of substitution strategies and systems of equations.

Based on the results of the questionnaire analysis given to 24 fifth-grade students, it was found that the students' responses to the use of the "Arithmetic Bridge" e-module were generally very positive. The questionnaire used consisted of closed-ended questionnaires using a Likert scale (1–5) and open-ended questions. Several aspects assessed included student engagement during learning, ease of understanding the material, impressions of the use of stories and images, interest

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in mathematical activities, and their perceptions of the level of difficulty of mathematics after using the e-module. Overall, students felt that learning with the e-module made them more engaged, easier to understand the concepts taught, and felt that mathematics became more enjoyable and no longer felt as difficult as before. The use of story contexts and visual illustrations was considered very helpful in bridging their understanding of basic algebraic concepts.

The results of this study indicate that the use of the “Jembatan Aritmatika” e-module, designed contextually and visually, can effectively improve students' understanding of the meaning of the equal sign and encourage the transition from arithmetic to early algebra. The shift in students' thinking, from direct calculation strategies to analyzing relationships between information, is a strong indication that this approach supports the development of relational thinking.

Factors that encourage the success of this implementation include: (1) the use of contexts that are close to students' everyday experiences, such as stories about shops, food, and animals; (2) concrete visualizations that make it easier for students to build mental representations; (3) step-by-step activities designed based on the results of the analysis of students' conceptual difficulties; and (4) the active involvement of students in exploring various possible solutions, which strengthens the conceptual understanding of the meaning of equations.

The didactic design approach used in this study allows for a systematic learning development process, starting with the analysis of real-world learning situations and continuing through implementation on both a small and large scale. Therefore, the developed e-module serves not only as a means of delivering material but also as a platform for students to explore and reflect deeply on mathematical concepts. However, this study was conducted with a limited sample size and within a single educational setting, so the results may not be broadly generalizable to a more diverse population.

The findings of this study support the results of Powell's (2012) which emphasized the importance of developing a relational understanding of the equals sign, and highlighted that many students of elementary level still have an operational understanding of the sign. Furthermore, this study aligns with Vermeulen and Meyer (2017) who showed that an explicit approach to the meaning of the equals sign is essential in the process of learning basic mathematics. The local context in this e-module also emphasizes the importance of culturally meaningful representations and students' experiences, as emphasized by Gravemeijer and Terwel (2000) in their approach to realistic mathematics education.

The theoretical implication of this research is that learning algebraic concepts does not have to start with formal symbols, but can be built gradually from concrete arithmetic experiences, through a didactic design process that considers meaning and context. Practically, the results of this study indicate that teachers can use the developed e-modules to increase student engagement and understanding of concepts previously considered difficult, such as equations and quantitative information systems. Furthermore, these results can be the basis for curriculum development or teacher training in designing more meaningful mathematics learning.

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CONCLUSIONS AND SUGGESTIONS

This study aimed to develop and implement the "Jembatan Aritmatika" e-module as a didactical bridge from arithmetic to early algebra, particularly by strengthening students' relational understanding of the equal sign. Based on the implementation and retrospective analysis, the study concludes that the e-module effectively supports students' conceptual development. Students demonstrated a significant improvement in interpreting the equal sign relationally, representing symbolic relationships, and solving simple equation problems. These results indicate that the didactical design—structured through a carefully developed learning trajectory—provides a meaningful contribution to reducing learning obstacles and supporting the transition from procedural arithmetic thinking to relational algebraic reasoning.

The findings of this study contribute to mathematics education by offering a validated didactical learning trajectory and an AI-assisted e-module that can serve as a prototype for bridging early algebra learning in elementary schools. As a recommendation, future research may expand the scope of the e-module to include topics such as variables, multi-step equations, or generalized arithmetic patterns. Further studies are also encouraged to test the module's effectiveness in diverse school contexts to strengthen its generalizability, as well as explore deeper integration of interactive technologies and formative assessments to enhance conceptual learning.

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