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MATHEMATICAL THINKING PROCESS OF STUDENTS IN SOLVING CONTROVERSIAL MATHEMATICS PROBLEMS BASED ON DECISION-MAKING TYPES

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Abstract

Developing thinking processes requires unusual treatment and stimulation, such as solving controversial mathematical problems. Problem-solving involves various stages of thinking and producing solutions based on decision-making. However, many students do not go through these stages perfectly, resulting in less optimal decision-making. This research aims to describe students' mathematical thinking processes in solving controversial problems based on decision-making types. The study uses a qualitative descriptive method by selecting 33 student subjects and grouping them based on decision-making types: empirical, heuristic, and rational. The grouping is based on scores from a decision-making type questionnaire, then 5 subjects are selected based on decision-making type, ability to answer questions, and mathematical communication. The instruments used in this study include a decision-making type questionnaire and controversial mathematical problems adapted from instruments developed by experts. Data is analyzed in three stages: data reduction, data presentation, and conclusion drawing. Empirical decision type (EM): Only successful up to the stages of generalizing and guessing, with difficulties in the convincing stage. Heuristic decision type (HE): Effective in specializing, generalizing, and guessing in the first problem but only meets two indicators in the convincing stage. Rational decision type (RA): Difficulty in the convincing stage and challenges in meeting the last stage indicators. The results of this study reveal the thinking process for each decision-making type when solving controversial mathematical problems. The thinking process holds significant importance, thus requiring special attention, especially from teachers, in monitoring students' thinking processes in the classroom.

Keywords: Controversial mathematical problems; decision-making types; mathematical thinking process.

Abstrak

Mengembangkan proses berpikir memerlukan perlakuan dan stimulasi yang tidak biasa, seperti memecahkan masalah matematika kontroversial. Pemecahan masalah mencakup berbagai tahap berpikir dan menghasilkan solusi berdasarkan pengambilan keputusan. Namun, banyak siswa tidak menjalani tahapan ini dengan sempurna, sehingga keputusan yang diambil kurang optimal. Penelitian ini bertujuan untuk mendeskripsikan proses berpikir matematis siswa dalam memecahkan masalah kontroversial berdasarkan tipe pengambilan keputusan. Penelitian ini menggunakan metode deskriptif kualitatif dengan memilih 33 siswa siswa dan dikelompokkan berdasarkan tipe pengambilan keputusan, yaitu empiris, heuristik, dan rasional. Pengelompokan tersebut berdasarkan skor pada kuesioner tipe pengambilan keputusan, kemudian dipilih 5 siswa berdasarkan tipe pengambilan keputusan, kemampuan menjawab soal dan komunikasi matematis. Instrumen dalam penelitian ini berupa kuesioner tipe pengambilan keputusan dan soal kontroversial matematis yang merupakan adaptasi dari instrumen yang dikembangkan oleh para ahli. Data dianalisis dalam tiga tahap, yaitu reduksi data, penyajian data, dan penarikan kesimpulan. Tipe pengambilan data empiris (EM): Hanya berhasil sampai pada tahap menggeneralisasi dan menduga, kesulitan pada tahap meyakinkan. Tipe pengambilan heuristik (HE): Efektif pada tahap mengkhususkan, menggeneralisasi, dan menduga pada soal pertama, namun hanya memenuhi dua indikator pada tahap meyakinkan. Tipe pengambilan keputusan rasional (RA): Kesulitan pada tahap meyakinkan dan kesulitan dalam memenuhi indikator tahap terakhir. Hasil penelitian ini

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mengungkapkan proses berpikir untuk setiap tipe pengambilan keputusan ketika menyelesaikan masalah matematika kontroversial. Proses berpikir memiliki arti penting, sehingga memerlukan perhatian khusus, terutama dari guru, dalam memonitor proses berpikir siswa di dalam kelas.

Kata kunci: *Masalah kontroversial matematis, proses berpikir matematis, tipe pengambilan keputusan.*

INTRODUCTION

Thinking is a fundamental prerequisite and the primary objective of learning. Essentially, thinking serves as the bridge between exploring new information and solving problems based on the acquired knowledge. Thinking is a multifaceted process with stages that include entry, exploration, and evaluation. Thinking as the cognitive process of generating ideas and strategies for problem-solving and decision-making (Utomo et al., 2023). Consequently, it's unfortunate if the thinking process isn't an integral part of teaching and learning activities. Thinking is a productive activity that directly yields solutions and, therefore, integrating thinking activities into every learning situation is essential for cultivating thinking skills (Tomé et al., 2019). Regrettably, the reality is that many students struggle with thinking, and some even avoid it entirely. Faced with problems, they often resort to procrastination, pretending to work, or copying others' solutions. This is particularly pronounced in mathematics, even though studying math is precisely the practice that sharpens thinking skills (Sriwongchai, 2015).

Developing the thinking process poses a challenge in education, requiring innovative approaches and stimuli. Assigning mathematical problem-solving tasks is one such approach (Andriani & Fraser, 2023). However, it's crucial that these problems meet specific criteria to foster quality thinking processes instead of worsening students' conditions. Mathematical problems should encompass key elements, including problem identification, analysis, evaluation of mathematical statements, generalization, visualization, and classification (Breen & O'Shea, 2019). Problem-solving is an avenue for students to enhance their thinking skills, making it a fundamental skill in math education (Rott et al., 2021). Solving problems through the right strategies and work order becomes a foundational ability in mathematical learning (Suprayo et al., 2023). Mathematical problems that stimulate the creativity of students are frequently not limited in scope and offer chances for students to discover innovative solutions, typically these issues manifest as having various potential solutions.

In this undertaking, we tackled a controversial problem that demands logical reasoning and diverse solutions. This strategy is based on the disputatious nature of the problem, requiring a rational approach and multiple methods of resolution. Mathematical controversial problems are known for generating differences of opinion and cognitive disputes(Subanji et al., 2021). These problems are non-routine and open to various interpretations (Subanji et al., 2023). Furthermore, they infuse energy into the learning process through classroom discussions and debates (Rosyadi et al., 2022). However, a teacher's role as a

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mediator is essential to maintain a conducive learning environment (Mueller & Yankelewitz, 2014). Solving controversial problems requires students to provide logical arguments and opinions(Rosyadi, 2021). This process enhances and develops students' thinking because these problems trigger cognitive conflicts (Susiswo et al., 2022) resulting from diverse perspectives on objects, concepts, and solution methods (Gusacov, 2022). So, it can be concluded that a mathematical controversial problem is a mathematical problem that is open-ended, allowing for differences of opinion and giving rise to arguments (Subanji et al., 2023). These differences of opinion can manifest in the form of concepts, procedures, or even results. Controversial problems exhibit distinctive characteristics and criteria, such as complexity in both content and solution procedure(Rosyadi et al., 2022; Subanji et al., 2021). Stresses that mathematical controversial problems must be high-level challenges involving robust argumentation (Rosyadi et al., 2021). Furthermore, these problems generally feature multiple solution strategies, encouraging students to enhance their mathematical abilities (Swastika et al., 2022) and thinking skills (Suryawan & Ratnaya, 2023). Based on the aforementioned statements, the characteristics and criteria of mathematical controversial problems are that they are highly complex, leading to arguments, and they offer various solution strategies to promote students' mathematical thinking processes.

Solving controversial problems requires an appropriate decision-making process, as decision-making is the most fundamental cognitive process humans possess (Murtafiah et al., 2021). Decision-making is a conscious activity used to select the best solution from a wide range of options (AlAli et al., 2023). Decision-making involves a series of actions, encompassing intuition, interaction, and analysis in problem-solving. The decision-making process is the thinking stage where the best idea among many is selected (Kämäräinen et al., 2021; Maulana & Rochintaniawati, 2021), ensuring that the decision taken is not misleading. In essence, the decision-making process is an individual's strategy for processing and organizing information to reach a problem's solution (Hafni & Nurlaelah, 2018; Lung-Guang, 2019). Considering the arguments presented, it can be concluded that decision-making skills are fundamental for solving problems, especially mathematical ones. However, students often do not follow the thinking process stages when solving mathematical problems, resulting in suboptimal decision-making. In many cases, students merely copy steps from previously solved problems (Liljedahl, 2021), indicating that problems provided do not promote perfecting the thinking process when solving mathematical problems. Decision-making style represents an individual's approach and reaction to problems (Orhan, 2022). Various typologies have been proposed, including categorized decision-making into four types: intuitive, heuristic, empiric, and rational, which closely resemble students' mathematical thinking processes when solving problems (Wang & Ruhe, 2007).

Research on mathematical thinking has been conducted previously by (Bintoro et al., 2022; Insan & Ni’mah, 2021; Khairunnisa et al., 2022; Sa’adah et al., 2023; Syam & Yunus, 2023). Mathematically controversial

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problems have also been studied by (Atmaja et al., 2023; Rosyadi et al., 2022; Subanji et al., 2023; Suryawan & Ratnaya, 2023; Swastika et al., 2022; Walida, 2022). However, decision-making styles have not been explored in the context of students' thought processes when solving mathematical controversial problems. Consequently, this research aims to investigate this topic, hoping to contribute a fresh perspective on the mathematical thinking process by integrating it with controversial problems and decision-making types.

Given the above explanations, this research will delve into the thought processes of junior high school students when solving mathematical controversial problems based on their decision-making styles. We will examine how students navigate the stages of mathematical thought, identifying selected criteria for solving controversial problems by considering their unique decision-making types.

METHODS

The research method used is descriptive qualitative research that aims to explain, step by step, students' thought processes when solving mathematical controversial problems based on their decision-making styles. For this study, the data sources consisted of junior high school students who were asked to complete questionnaires and written tests, with observations made to observe students' thought processes when answering the problems. At the initial stage, 33 students completed a decision-making type questionnaire adapted from the taxonomy of decision-making strategies and types developed by (Wang & Ruhe, 2007). This questionnaire was used to identify students' decision-making styles or categories. Furthermore, 33 students were categorized into 3 decision-making types such as empirical, heuristic, and rational, based on the largest score obtained. Next, students were instructed to solve math controversial problems. These controversial problems are presented in a descriptive format, covering multidimensional aspects, thus eliciting various perspectives, including those related to definitions, axioms, theorems, solution methods, and other mathematical concepts (Simic-Muller et al., 2015). The problems were adapted from the works of (Suryawan & Ratnaya, 2023; Swastika et al., 2022; Walida, 2022) then observations were made during the problem solving process. Then 5 subjects representing different decision-making types were selected based on the results of the observations by considering the mathematical ability seen from the results of working on controversial problems, then mathematical communication to make it easier for researchers to explore the information sought.

The selection of research subjects was conducted iteratively and continuously until the collected data reached a saturation point, indicating a consistent pattern across multiple research subjects. Based on the data saturation, five research subjects were chosen, specifically EM 1, EM 2, HE 1, HE 2, and RA, all exhibiting empirical, heuristic, and rational decision-making styles. Subject selection involved several factors, considering various aspects (Sugiyono, 2018), aligned with the research objectives (Ahyar et al., 2020). These considerations encompassed decision-making styles, data saturation, students' problem-solving abilities, oral communication proficiency, input from mathematics teachers

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(Suharsimi, 2013). However, for this study, subjects with high mathematical thinking abilities were chosen to ensure comparability, adhering to the following criteria for mathematical ability levels:

Data analysis techniques in this study include: (1) data reduction, namely reducing data which includes the results of student work complemented by the results of observations and in-depth interview results which are realized in the form of verbal expressions of mathematical thinking processes when solving controversial problems based on the type of decision making. The results of students' mathematical thinking processes in solving contextual problems were then categorized according to the stages of students' problem solving processes and their respective decision-making types. Drawing verification/conclusion is the last step. (2) Data presentation includes analyzing and describing data as a reference for drawing conclusions from the research conducted. (3) Withdrawing verification/conclusion.

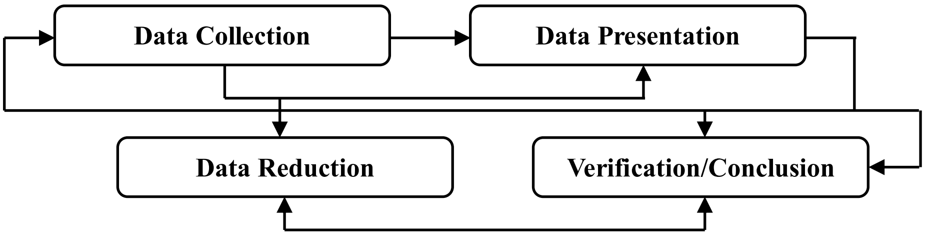

Figure 1. The data analysis

In this study, the model for the mathematical thinking process is adapted from the framework proposed by (Stacey, 2011). Several descriptors have been drawn from (Çelik & Özdemir, 2020; Tohir et al., 2020; Tsang, 2014; Yildiz, 2016). The following outlines the stages of the mathematical thinking process along with their respective indicators.

The types of decision making, as categorized by (Wang & Ruhe, 2007), have been adapted for this study. The intuitive aspect has been excluded from the model, as it is perceived to be less suitable for solving scientific problems (Winarso, 2014).

RESULTS AND DISCUSSION

This research was carried out in one of the junior high schools in Kuningan during the 2022/2023 school year. The data presentation and analysis in this section present the results of the work completed by high-achieving students, along with interviews conducted to observe their mathematical thinking processes at various stages when solving controversial problems. These problems have been adapted from those developed by (Suryawan & Ratnaya, 2023; Swastika et al., 2022; Walida, 2022). They have undergone validation and practicality testing to reveal their potential impact in this research, particularly in relation to the types of decision making defined by (Wang & Ruhe, 2007). The selected subjects are detailed in the Table 1.

Table 1. The selected subjects.

No	Code	Decision-Making Type
1	EM1	Empirical 1
2	EM2	Empirical 2
3	HE1	Heuristic 1
4	HE2	Heuristic 2
5	RA	Rational

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This section describes the data regarding the results of the subjects' work in solving controversial problems. Here are the results of EM 1's work (Figure 1) in solving controversial math problem.

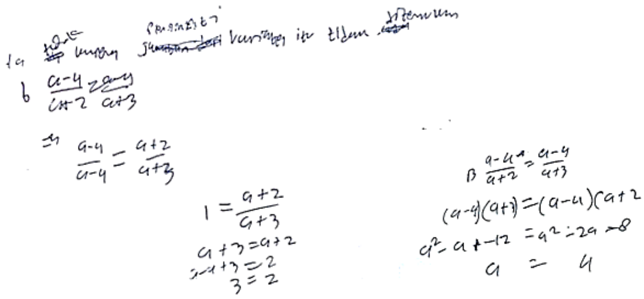

Figure 2. The EM 1's work

EM 1's work demonstrates the correct answer in solving the controversial problem. EM 1 exhibits a strong understanding of the concept for solving rational equations. Additionally, EM has the ability to perform algebraic operations to find the desired variable value, making the resolution of controversial math problems relatively effortless.

Specializing (SP)

When considering the indicators of the mathematical thinking process stages in the specializing aspect, it is apparent that EM 1 can identify the problem by expressing disagreement regarding the inability to find the variable value. Furthermore, EM 1 identifies an error in the solution process by arriving at the incorrect result of 3 = 2. Moreover, EM 1 demonstrates a potential solution strategy by attempting algebraic operations on the rational equation, aiming to determine the variable value as a = 4.

Generalizing (GN)

At the generalizing stage, EM 1 reflects on their ideas by recognizing errors, considering and reevaluating their concepts. This is illustrated through the process of algebraic multiplication operations undertaken by EM 1. Additionally, EM 1 broadens the scope of their results, involving the concept of algebraic operations and implicitly suggesting that the outcome of the equation typically results in finding the value of a variable.

Conjecturing (CJ)

In the Conjecturing stage, EM 1 demonstrates the ability to draw analogies between the current problem and similar concepts encountered previously. This can be observed in the solution strategy employed by EM 1, indicating a grasp of the solution method for the controversial problem at hand.

Convincing (CV)

As for the Convincing aspect, EM 1 provides reasoning for the answer given by following the necessary steps to attain the desired solution. Although some indicators are yet to be clearly visible.

The results of EM 2's answers in solving controversial math problem (Figure 3).

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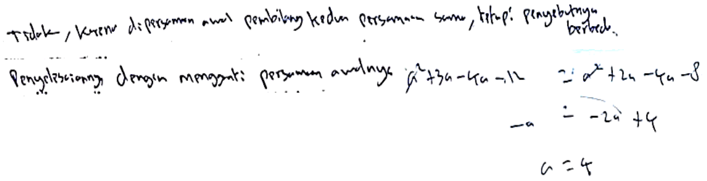

Figure 3. The EM 2's work

EM 2's response successfully yielded the correct result in solving the aforementioned controversial problem, employing the method of solving rational equations through algebraic operations.

Specializing (SP)

At this stage, EM 2 identifies the problem by expressing disagreement with the answer to the problem, followed by a well-explained rationale. Subsequently, EM 2 elucidates the errors present in the problem, such as the commonality of numerators in the two segments, contrasted with differing denominators. The subject then outlines the appropriate strategy for solving mathematically controversial problems to derive the variable value, a = 4.

Generalizing (GN)

EM 2 engages in reflection regarding the strategy employed, discerning problem errors, and adapting the strategy to the specific problem and existing knowledge. Moreover, EM 2 endeavors to incorporate relevant concepts.

Conjecturing (CJ)

In the Conjecturing aspect, EM 2 displays the ability to draw analogies between the current problem and previously encountered problems and knowledge, simplifying the provided answers.

Convincing (CV)

The subject, EM 2, offers an explanation of the obtained answer but does not exhibit the formation of a pattern or its inverse. The work of HE 1 on controversial problem is shown in the Figure 4.

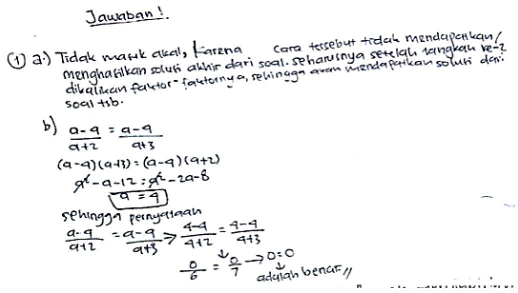

Figure 4. The HE 1's work

HE 1 arrived at the correct answer while employing multiple reasons and strategies necessary for solving the controversial problem. HE 1 also made efforts to substantiate the answer by substituting variable values into the equation presented in the problem.

Specializing (SP)

HE 1 detected issues with the problem's answer and expressed disagreement, providing a rationale for this perspective. Subsequently, HE 1 offered an illustrative explanation of the issue within the problem. Additionally, HE 1 proposed an alternative solution strategy distinct from the problem's answer.

Generalizing (GN)

HE 1 conducted checks that pointed to the identification of errors and suggested alternative strategies for

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solving the problem. Furthermore, HE 1 engaged with a wide range of concepts while addressing the controversial problem.

Conjecturing (CJ)

HE 1 proceeded to solve the problem by leveraging their existing knowledge, resulting in a commendable solution. This proficiency is reflected in the systematic steps undertaken by the subject.

Convincing (CV)

In the final stage, HE 1 aimed to clarify the answers obtained. Moreover, HE 1 rechecked their work by substituting variable values into the equation, ultimately arriving at the equation 0 = 0. This marks the formation of a pattern based on the results obtained. The work of HE 2 on controversial problem is in Figure 5.

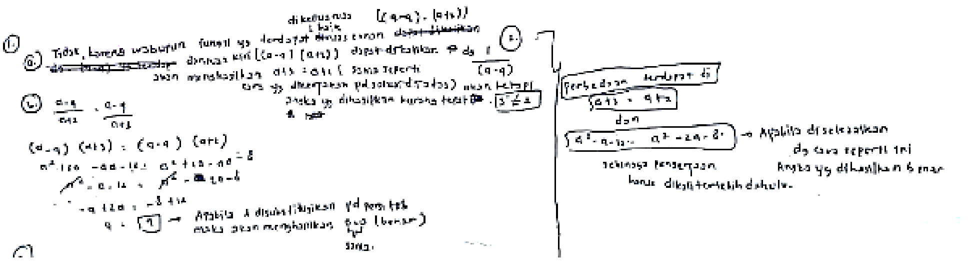

Figure 5. The HE 2's work

HE 2 achieved a perfect solution, marked by a well-executed process encompassing reasoning, strategy formulation, and solution methods. Consequently, HE 2 arrived at the desired answer, supported by several rational opinions.

Specializing (SP)

In the context of the specializing stage of the thinking process, HE 2 expressed disagreement with the provided answer, signifying the identification of issues within the problem. HE 2 also illustrated the problem's discrepancies, highlighting the irrationality of the proposed solution. For example, $a - 4a + 2 = a - 4a + 3$ can be resolved by multiplying by $1a - 4$, yielding $a + 3 = a + 2$, as presented in the problem. However, $3 \neq 2$. Subsequently, HE 2 attempted to propose a potentially correct strategy, starting with algebraic multiplication operations to attain a quadratic equation form in both segments.

Generalizing (GN)

HE 2 considered prospective solutions as a manifestation of reflective thinking and prudence to avoid incorrect answers. Furthermore, the subject sought to broaden the scope of ideas, implicitly suggesting that both segments should equate to reveal the accurate value.

Conjecturing (CJ)

HE 2 engaged in a process of analogizing problems previously encountered with the present challenge. This is evident in the manner in which the subject provided insights into the anomalies within the current problem. Ultimately, HE 2 effectively solved the problem, showcasing their abilities and prior knowledge.

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Convincing (CV

In the final stage, HE 2 elucidated the answers, commencing with the reasons and sound solution methods required to obtain the correct answer. Additionally, HE 2 constructed a pattern from the existing rational equation to reinforce their arguments, achieved through the substitution of variable values resulting in the equation 0=0, which is accurate due to the matching values.

Furthermore, the work done by RA in solving controversial problem. RA's response was highly commendable, as evident from the inclusion of multiple rationale and a well-structured process to yield the accurate solution. Additionally, RA's approach to the task demonstrated a strong communicative element, facilitating the elucidation of the answer.

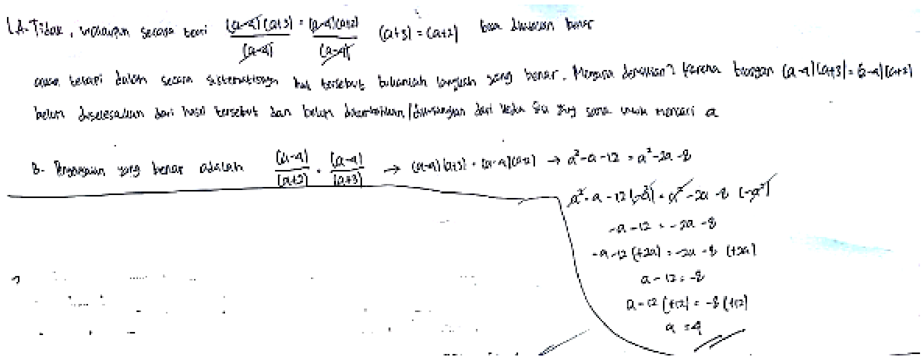

Figure 6. The RA's work

Specializing (SP)

In the initial stage, RA voiced dissent regarding the problem's solution, offering a unique perspective that underscored RA's problem-identification process. Furthermore, RA provided a detailed explanation of the error, pinpointing its location while simultaneously outlining the correct procedure.

Generalizing (GN)

RA attempted a reevaluation, noting that the problem's answer might be theoretically plausible but incorrect based on systematic reasoning. RA also shared insights regarding the breadth of ideas required to address problems related to rational equations, emphasizing that finding the value of variable 'a' necessitates a series of subtraction or addition operations.

Conjecturing (CJ)

RA's conjectures exhibited a keen analogical process, drawing on prior knowledge to relate to the present problem. This underscored RA's proficiency and familiarity with similar problem types.

Convincing (CV)

RA strived to tackle the problem using their existing strategies, effectively articulating the reasons behind the answers provided. RA's response to the controversial problems was noteworthy for its clarity and adaptness.

Based on research on students' mathematical thinking processes in solving controversial problems, empirical students (EM1 and EM2) collect relevant data and information, preferring concrete information such as

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case examples and numbers. They learn through real-life experiences and often use trial and error when facing new problems, as stated by (Facione & Facione, 2007). Empirical students tend to be less independent and frequently engage in discussions or ask questions, as they rely on experience and require validation, as highlighted by (Wang & Ruhe, 2007). Empirical students have a more structured thinking process, avoiding hasty conjectures and using multiple strategies to solve problems. They are also able to better explain the reasoning behind their answers (Khairunnisa et al., 2022). This suggests that concept understanding heavily depends on students' experiences. Therefore, the role of the teacher is crucial in designing project-based learning, case studies, group discussions, and providing feedback and access to learning resources to encourage empirical students to build confidence in problem-solving.

Heuristic students (HE1 and HE2) aim to understand the information provided by identifying known facts and the question at hand, then linking them to their existing knowledge to find a solution. According to (Ghazal, 2014; Wang & Ruhe, 2007) heuristic students rely heavily on concepts and knowledge to solve problems. After understanding the problem, they explore various strategies and actively evaluate their effectiveness, often using multiple strategies rather than just one, as noted by (Newton et al., 2022). Heuristic students typically solve problems using established concepts or rules, which allows their solutions to be formally proven, though they are highly dependent on their existing knowledge (Yusuf & Ekawati, 2020).

Their thinking process is mature, demonstrated by careful decision-making and well-explained reasoning, both verbally and through calculations. This flexible and adaptive approach enables heuristic students to solve mathematical problems effectively (Insan & Ni'mah, 2021). However, the teacher's role remains essential in guiding the development of heuristic thinking skills.

When faced with controversial mathematical problems, rational decision-making students (RA) begin by thoroughly understanding the problem. They read the question carefully, identify the given information, the question itself, and what needs to be solved. This approach aligns with, who state that rational students are more systematic in problem-solving. Rational students are also efficient in answering questions without sacrificing accuracy (Murtafiah et al., 2021) paying attention to the efficiency of their responses.

These students are able to visualize problems and use mathematical symbols effectively, as confirmed by (Hakim et al., 2022), who found that rational students excel in mathematical visualization and symbolism. Despite their efficiency, rational students are cautious when providing answers, ensuring their decisions are well-considered (Eisenführ et al., 2010). Unlike intuitive students, rational students give thorough, structured, and systematic answers, as seen in their work on controversial problems. According to (Suwarto et al., 2023), they focus on structure, ensuring systematic responses.

Rational students demonstrate solid mathematical thinking throughout the process, including specializing, generalizing, conjecturing, and convincing, all with careful consideration (Wang & Ruhe, 2007). This makes the rational decision-making type ideal for

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helping students understand and solve problems. However, achieving this type requires teacher support in designing lessons to develop rational decision-making skills. Relevant instructional strategies for rational students include providing access to comprehensive information, facilitating group discussions for idea exchange, and encouraging logical reasoning and justification in problem-solving, as rational students need example information for deep analysis(Putri & Haerudin, 2020).

CONCLUSIONS AND SUGGESTIONS

This study aimed to describe students' mathematical thinking processes in solving controversial problems based on dechafniision-making types. The results revealed that empirical decision-making types (EM) successfully executed the specializing, generalizing, and conjecturing stages but encountered difficulties in the convincing stage. Heuristic decision-making types (HE) were effective in specializing, generalizing, and conjecturing but faced challenges in the convincing stage, especially in the second problem. Rational decision-making types (RA) experienced difficulties in the final convincing stage in both the first and second problems.

This study confirms that there is no universally correct or incorrect type of decision-making; its effectiveness depends on the context and complexity of the problems faced. Recommendations: Teachers should pay special attention to students' mathematical thinking processes, especially in guiding them through optimal thinking stages. Provide exercises that encompass various decision-making types to develop students' mathematical thinking skills. Conduct continuous assessments of students' mathematical thinking abilities to identify areas needing improvement. Teachers need to develop teaching materials that encourage students to go through each thinking stage, particularly the convincing stage. With dedicated attention from teachers and the right approach, students can enhance their mathematical thinking abilities and make more optimal decisions in problem-solving.

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