

DISSECTING STUDENT MISCONSTRUCTION IN TRANSFORMATIONAL ACTIVITIES SOLVES PROBLEMS THAT ALLOW COGNITIVE CONFLICT TO OCCUR

Rosimanidar¹, Purwanto², Erry Hidayanto^{3*}, I Made Sulandra⁴

¹Department of Mathematics Education, Institut Agama Islam Negeri (IAIN)
Lhokseumawe, Aceh, Indonesia

^{1,2,3,4}Department of Mathematics Education, Universitas Negeri Malang, Malang, Indonesia

*Corresponding author. Jl. Semarang No. 5 (B24 FMIPA UM Building), Malang, Jawa Timur, Indonesia

E-mail: rosi@iainlhokseumawe.ac.id¹⁾
purwanto.fmipa@um.ac.id²⁾
erry.hidayanto.fmipa@um.ac.id^{3*)}
made.sulandra.fmipa@um.ac.id⁴⁾

Received 08 January 2024; Received in revised form 07 March 2024; Accepted 19 September 2024

Abstract

The transformative activity was crucial for problem-solving. In resolving problems, cognitive conflicts could arise, characterized by construction errors marked by deviations or differences from scientific concepts. As many as 106 students experienced cognitive conflict, and eight made construction errors in transformational activities. This research aimed to describe students' construction errors in transformative activities when solving problems that might lead to cognitive conflicts. The research design was phenomenological, with four subjects selected and one student from each group. Student responses and interview results served as research data, analyzed through narrative text analysis. The research findings revealed four forms of construction errors in students during transformative activities in problem-solving that might lead to cognitive conflicts: (1) pseudo construction "correct," occurring when students provide a correct answer to a problem, but upon closer examination, it was found that the clarification of the answer was incorrect; (2) pseudo construction "incorrect," happening when students gave an incorrect answer to a problem, but upon closer examination, the students had a correct thought process and could provide the right answer; (3) hole construction errors, occurring when there were inconsistencies in the construction process of concepts in students' minds; and (4) mis-analogical construction errors, occurring when students made errors in analogizing a problem with representations of other concepts. These four construction errors occurred in transformative activities based on incomplete rule-based systems. Examining these construction errors allowed instructors to improve students' transformative thinking activities according to linear equations with one variable.

Keywords: construction error; cognitive conflict; procedural trap; transformational activities

Abstrak

Aktivitas transformasional sangat penting dalam menyelesaikan masalah. Dalam penyelesaian masalah kemungkinan terjadi konflik kognitif, yaitu kesalahan konstruksi yang ditandai ada penyimpangan atau perbedaan dengan konsep ilmiah. Mahasiswa mengalami konflik kognitif sebanyak 106 orang dan 8 mahasiswanya telah melakukan kesalahan konstruksi dalam aktivitas transformasional. Penelitian ini bertujuan untuk mendeskripsikan kesalahan konstruksi mahasiswa dalam aktivitas transformasional menyelesaikan masalah yang memungkinkan terjadi konflik kognitif. Jenis penelitian ini adalah fenomenologi dengan dipilih 4 subjek penelitian yang masing-masing 1 mahasiswa dari setiap kelompok tersebut. Jawaban mahasiswa dan hasil wawancara digunakan sebagai data penelitian. Data penelitian dianalisis melalui analisis teks naratif. Temuan penelitian diperoleh bahwa ada empat bentuk kesalahan konstruksi mahasiswa yang dibedah dalam aktivitas transformasional menyelesaikan masalah yang memungkinkan terjadi konflik kognitif, yaitu (1) pseudo construction "benar" yang terjadi saat mahasiswa memberikan jawaban benar terhadap suatu permasalahan, namun ketika ditelusuri, ternyata mahasiswa salah dalam memberikan klarifikasi jawaban; (2) Kesalahan pseudo construction "salah" yang terjadi saat mahasiswa memberikan jawaban salah terhadap suatu permasalahan, namun ketika ditelusuri mahasiswa mempunyai cara berpikir yang benar dan dapat memberikan jawaban yang benar;

DOI: <https://doi.org/10.24127/ajpm.v13i3.9457>

(3) Kesalahan lubang konstruksi (*hole construction*) yang terjadi saat proses konstruksi konsep dalam pikiran mahasiswa ada yang tidak sesuai; dan (4) kesalahan mis-analogical construction yang terjadi saat mahasiswa membuat kesalahan dalam menganalogikan masalah dengan representasi konsep lain. Keempat kesalahan konstruksi tersebut terjadi pada aktivitas transformasional berbasis aturan tidak lengkap (*incomplete rule-based*). Penelaahan kesalahan konstruksi ini dijadikan dasar bagi dosen untuk memperbaiki aktivitas berpikir transformasional mahasiswa sesuai konsep persamaan linier satu variabel

Kata kunci: aktivitas transformasional; kesalahan konstruksi; konflik kognitif; perangkap prosedural



This is an open access article under the [Creative Commons Attribution 4.0 International License](https://creativecommons.org/licenses/by/4.0/)

INTRODUCTION

Research on problem-solving that triggers cognitive conflict is a critical issue to aid learners in transforming their cognitive thinking activities. For instance, Fujii (1987) identified three cognitive conflicts: (1) C1: conflict in the 'behaviorized how'; (2) C2: conflict in the verbalized how'; and (3) C3: conflict in the 'justification of how,' in transforming learners' understanding from instrumental to relational comprehension. Fraser (2007) found signs of cognitive conflict to include uncertainty and confusion when a learner recognizes anomalies conflicting with their expectations; doubt and reevaluation of the situation to try and resolve the conflict; a passionate curiosity and high interest in the problem; and tension, frustration, and anxiety when encountering questions more challenging to solve than expected. Interventions are then implemented to address cognitive conflict situations, strengthening learners' structural and procedural understanding of algebra. Tirosh & Graeber (1990) have discovered that effective cognitive conflict situations transform procedural knowledge into structural conceptions of division. Fujii (2003) developed cognitive conflict problems to acquire and investigate Japanese learners' understanding of algebra. Halimah et al (2019) stated that male learners more frequently experienced cognitive

conflict compared to female learners, and learners in the learning process were advised not only to memorize formulas but also to understand the concepts deeply. Pratiwi et al (2019) found two characteristics of learners' cognitive conflict in problem-solving from the perspective of information processing theory, namely (1) error—cognitive conflict—equilibrium—solving the problem and (2) error—cognitive conflict—equilibrium—couldn't solve the problem. Maharani & Subanji (2018) mentioned that cognitive conflict could be resolved with scaffolding in algebra to guide learners' prior knowledge and strengthen their understanding. Several studies indicated a change in learners' concepts by examining errors in solving problems that might lead to cognitive conflict (Irawati et al., 2018; Maharani & Subanji, 2018; Pratiwi et al., 2022; Sela & Zaslavsky, 2007; Setiawan et al., 2023; Sutopo, 2014; Walida et al., 2022; Wyrasti, Sa'Dijah, et al., 2018; Zazkis & Chernoff, 2008).

Some learners continue to make errors when solving problems that could result in cognitive conflict. Learners typically need help building the concept of algebraic addition in the form of roots, misplacement of concepts, pseudo-thinking, and misanalogy (Setiawan et al., 2023). According to Wyrasti et al (Wyrasti, Sa'Dijah, et al., 2018), completing tasks involves misanalogous constructions in set theory, fractions, and

DOI: <https://doi.org/10.24127/ajpm.v13i3.9457>

sequences. Learners make errors in solving problem-solving question forms (Sutopo, 2014). According to Pratiwi et al (Pratiwi et al., 2022), routine errors in elementary school teacher education students occur when solving problems with the same denominator fractions—errors in understanding unresolved problems (Walida et al., 2022). Errors in determining the solution to linear equations with one variable and linear inequalities with one variable designed to eliminate x (the disappearance of x) are due to a need to understand its meaning (Fraser, 2007; Fujii, 2003; Fujii, 1987). These are part of construction, conceptual, and procedural errors. In this paper, the researcher focuses on construction errors in transformational activities because many learners still need to improve (Badawi et al., 2016) and get stuck in transformational work (Coles & Brown, 1998).

Transformational activity is one of the algebraic thinking activities. Algebraic thinking is a mental process of thinking in humans involving generational, transformational, and meta-global level activities, both related to algebra and other than algebra. Transformational activity here refers to changing the algebraic form or equations based on rules to maintain equivalence (Kieran, 2004). In this process of change, procedural steps are needed based on rules. Procedural steps to solve linear equations with one variable include writing the equation, using the distributive property and equality property, simplifying, and determining the solution (Malloy et al., 2002).

Then, transformational activity in solving problems that could lead to cognitive conflict was observed in several problems related to equations and inequalities. Fraser (Fraser, 2007) presented problems with questions and new information containing procedural traps.

According to Sela (2008), contradictory problems were provided to challenge learners' formal and procedural knowledge, referred to as challenging procedural understanding. Similarly, according to Limón (2001), analogies and metaphors were used in discussions with friends or groups. In this research, the focus was limited to procedural trap problems because this type still caused some students to make construction errors. Numerous studies- claimed issues with procedural trap types could result in cognitive conflict and errors in their resolution supported this (Fraser, 2007; Sari, 2021). The possibility of conflict and the errors experienced were due to students being accustomed to solving problems according to the procedures they knew without identifying the relational elements formed by the equation.

The type of procedural traps problem allows cognitive conflict to occur, triggered by questions and new information provided involving questions that contain procedural traps. This is a point in procedural solutions where the procedures taught to students lead to odd results. This odd result can be solved if students master the understanding of a process. There needs to be a rote procedure to solve the problem. Students enter a realm outside standard procedures and must connect anomalous outcomes to the underlying structure of a conflict.

The benefits of transformational activities in resolving cognitive conflict problems with procedural traps help students master solving linear equations with one variable. Then, problem-solving can assist students in the most effective algebraic thinking activities (Booker & Windsor, 2010). Learners possess good critical thinking skills in learning (Maharani & Subanji, 2018). The change in learners' understanding of equations from memorizing concepts without

DOI: <https://doi.org/10.24127/ajpm.v13i3.9457>

connecting them to someone using a mathematical procedure comes from linking various relevant mathematical concepts in solving a problem and knowing why the procedure can be used (knowing what to do and why) (Fraser, 2007; Fujii, 1987, 2003).

However, for students in Indonesia, engaging in transformational activities to resolve cognitive conflict problems has yet to reach such changes, as they still experience construction errors in the solution procedures and the potential occurrence of cognitive conflicts. The researcher noticed this phenomenon in a preliminary investigation of the Mathematics Education program at UIN Maulana Malik Ibrahim Malang in East Java during the odd semester of 2022–2023. A total of 15 students experienced cognitive conflict, 3 students did not experience cognitive conflict, and 1 student avoided cognitive conflict. Among the

students experiencing cognitive conflict, 2 students made construction errors in the procedural stage during transformational activities, with mis-analogical construction, and 1 made a hole construction error. Subanji (2015) states that mis-analogical construction is a construction error caused by students equating one concept with another. Meanwhile, hole construction is a construction error experienced by students due to an incomplete thought structure formed during the concept construction process. Forms of construction errors in mathematical concepts include (1) pseudo construction, (2) hole construction, (3) mis-analogical construction, and (4) mis-logical construction (Inganah et al., 2021; Ni & Halim Fathani, 2018; Putri & Indrawatiningsih, 2023; Subanji, 2015). Here is an example of a student experiencing a misanalogical construction error.

Tentukan penyelesaian dari

(a)

$$\frac{1}{3}(12-6x) = 4-2x$$

$$12-6x = 12-6x$$

$$12-12 = -6x+6x$$

$$0 = 0$$

(b)

3. Jelaskan langkah-langkah penyelesaiannya?

1. mengalikan 3 agar angka yang pecahan hilang
2. mengelompokkan angka yg bervariasi dengan variabel dan konstanta dg konstanta
3. kemudian di kurangkan maka hasilnya 0

$$3 \cdot \frac{1}{3}(12-6x) = 3(4-2x)$$

$$12-6x = 12-6x$$

$$12-12 = 6x-6x$$

Figure 1. Student Answers Ch

In Figure 1, it is observed that students engage in transformational activities with stages such as not writing equations, using distributive properties, not employing equality properties, simplifying, and not determining

solutions. These issues prompt researchers to investigate student construction errors in problem-solving that may lead to the aforementioned cognitive conflicts, which are worthy of further exploration. Several studies on

DOI: <https://doi.org/10.24127/ajpm.v13i3.9457>

construction errors in problem-solving based on cognitive conflict have been conducted (Pratiwi et al., 2019; Setiawan et al., 2023; Sutopo, 2014; Wyrasti et al., 2018). However, there has been no research on construction errors in transformational activities in problem-solving that may lead to cognitive conflicts in the context of linear equations with one variable. This study aims to describe students' construction errors in transformational activities when solving problems that may lead to cognitive conflicts. It is hoped that this research will be beneficial in identifying student construction errors, thus aiding in improving students' transformational thinking activities through the concept of linear equations with one variable.

METHODS

The emerging phenomenon in this study is to be explored by the researcher. Therefore, this research falls under the qualitative approach with a phenomenological type. This phenomenological research analyzes and describes the individual processes of students when experiencing construction errors in transformational activities to solve cognitive conflict problems, wherein the researcher aims to construct the essence of the student's experiences. This statement about phenomenological research, as stated by (Creswell & Creswell, 2018), thus becomes an understanding of the unique nature of something (Creswell, 2014). The study was conducted in the odd semester of 2023–2024, involving 143 students from five universities in Indonesia. These universities included the Department of Mathematics Education or Mathematics Education study programs at IAIN Lhokseumawe and IAIN Takengon in Aceh, UIN SATU Tulungagung in East Java, Universitas Lambung Mangkurat in South Kalimantan, and

STKIP YDB Lubuk Alung in West Sumatra. of the 143 respondents, 10 data points were considered errors due to the given questions' dual meanings (ambiguity). The exclamation mark (!) at the end of the question was assumed to mean factorial, and a number was incorrectly written in the question as "15" instead of "5". Therefore, the data were reduced to 133 respondents. The students had completed calculus or elementary algebra courses.

The research instruments consisted of a test about cognitive conflict problems in linear equations with one variable using procedural traps, as presented in Figure 2, and an interview guide. The cognitive conflict-based algebra test was used to solve problems with cognitive conflict in linear equations with one variable that had construction errors during transformational activities. Meanwhile, the interview guide was used to clarify the obtained data and identify the alignment between students' written answers and oral explanations in solving cognitive conflict problems. Of the 133 respondents, 106 experienced cognitive conflicts, while 27 did not. Among the 106 respondents who made construction errors, eight potential subjects were identified, comprising one potential subject with the error of pseudo construction "correct," 1 potential subject with pseudo construction "incorrect," 4 potential subjects with hole construction errors, and two potential subjects with mis-analogical construction errors. These errors were observed in the complete and incomplete transformational activity answers.

Problem 1

Determine the solution of

Problem 2

Determine the solution of

$$2x - 15 = 2(x - 4)$$

Figure 2. The problem of “cognitive conflict”

Data was collected from answer sheets and interview results by documenting construction errors in transformational activities to solve cognitive conflict problems. Based on data from prospective research subjects, four forms of construction errors were identified: *pseudo construction* “correct”, *pseudo construction* “incorrect”, *hole construction*, and *mis-analogical construction*. Research subjects were selected using purposive sampling, considering prospective subjects could communicate effectively when asked for further clarification regarding the completed work process. Four subjects were chosen for this study based on this selection method. The research subjects and the construction errors they made are as follows: Ak experienced the *pseudo-construction* “correct” error. Sar experienced the *pseudo construction*

“incorrect” error. YRH represented the *hole construction* error group. AWH represented the *mis-analogical construction* error group. The collected data, in answer sheets and interview transcripts, were validated using triangulation methods. Subsequently, the data were analyzed using qualitative analysis techniques, specifically narrative text analysis (Walida et al., 2022). The final stage involved concluding the research findings as achievements towards the objectives.

RESULTS AND DISCUSSION

In this study, the findings revealed construction errors made by students in transformational activities to solve problems that might have led to cognitive conflict. These errors were derived from analyzing answer sheets and interview results with the subjects through narrative text analysis. Based on the data analysis for subjects resolving cognitive conflict problems, subject Ak made construction errors in transformational activities, as depicted in Figure 3.

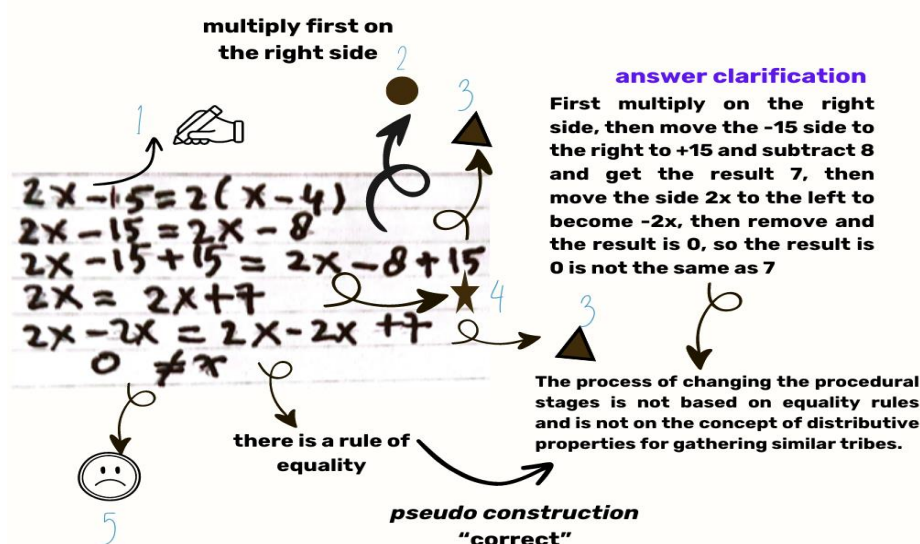


Figure 3. Construction Errors in Ak Subject Transformational Activities to Resolve Cognitive Conflict Problems

DOI: <https://doi.org/10.24127/ajpm.v13i3.9457>

Figure 3 shows that subject Ak worked on it using the rules of transformational activities with steps but did not determine the result of solving the equation. The procedural steps included writing the equation, using the distributive property, applying equality, simplifying, and not resolving the solution. The student correctly responded to a problem using the equality rule in the answer, but a construction error occurred. This construction error happened because, upon further investigation through an interview, it was revealed that the student was errors in clarifying the answer. The student answered that the procedural changes were not based on the equality rule and were not yet in line with the

concept of distributive property to collect like terms. This situation illustrates that subject Ak made a pseudo-construction "correct" error. Therefore, it can be concluded that subject Ak made a pseudo-construction "correct" error in transformational activities based on an incomplete rule-based system to solve problems that may lead to cognitive conflict. In addition to the "correct" pseudo-construction error, there was also a "wrong" pseudo-construction error in other subjects. In Figure 4, subject Sar made a "wrong" pseudo-construction error in transformational activities to solve problems that may lead to cognitive conflict.

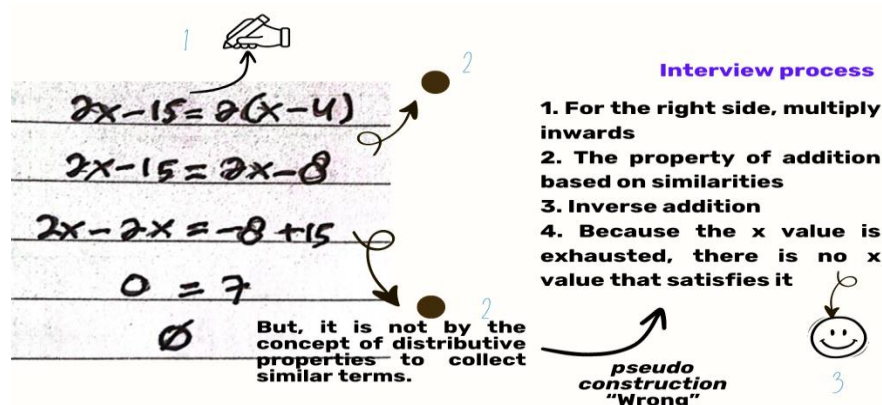


Figure 4. Construction Errors in Sar Subject's Transformational Activities for Resolving Cognitive Conflict Problems

The results from Figure 4 show that subject Sar worked on it using the rules of transformational activities but still needs to complete it. The procedural steps included writing the equation, using the distributive property, and not determining the solution. The answer sheet did not show the conclusion of the results of solving the equation. However, after confirmation in the interview process, the solution could already be inferred, and more procedural steps were

taken, including using the distributive property, applying equality, and determining the solution. On the answer sheet, the student responded incorrectly to a problem that needed to align with the concept of distributive property to collect like terms. However, upon investigation during the interview, it was found that the student had the correct thought process and could provide the right answer. Therefore, it can be concluded that subject Sar made a

DOI: <https://doi.org/10.24127/ajpm.v13i3.9457>

"wrong" pseudo-construction error in transformational activities based on an incomplete rule-based system to solve problems that may lead to cognitive conflict.

The term "pseudo construction" is highly appropriate for dissecting students' thinking errors, such as those observed in the transformational activities conducted in this study. This analysis can be carried out by examining the pseudo-thinking process. According to Subanji & Nusantara (2013), one way to identify thinking errors is to look into how students construct mathematical concepts using pseudo-thinking. The pseudo-thinking process in both "correct" and "incorrect" pseudo-constructions is spontaneous, rapid, and unconscious. This assertion aligns with statements made by several researchers, emphasizing that the pseudo-thinking process is intuitive, rapid, and

unconscious (Leron & Hazzan, 2009; Pape, 2004; Vinner, 1997). In the case of "incorrect" pseudo-construction errors, students could correct their answers, as seen in the interview results. The interview results constitute a part of the reflection activity for converting students' answers. As researcher Anggraini et al (2018) mentioned, students can correct their answers during reflection; in other words, students engage in false pseudo-thinking. Similarly, pseudo-thinking can be refined and improved through reflection. Imam & Kusmaryono (2023) support this claim by claiming that scaffolding and reflection can improve and refine pseudo-thinking.

Then, other construction errors also appeared in subject YRH in the transformational activity of resolving cognitive conflict problems, as shown in Figure 5.

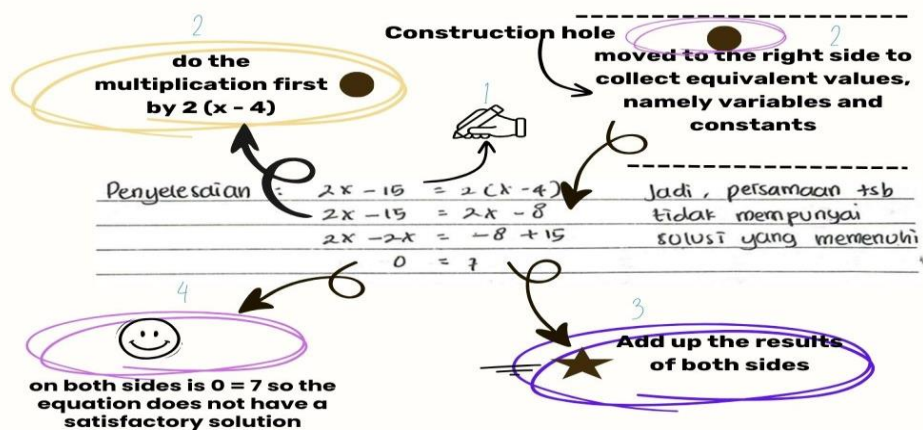


Figure 5. Construction Errors in YRH Subjects' Transformational Activities to Resolve Cognitive Conflict Problems

Figure 5 shows that subject YRH used transformational activities to solve cognitive conflict problems. These activities included writing the equation, using the distributive property, making it easier to understand, and finding the answer. Here, the activity was based on

incomplete rule-based procedures. For subject YRH, *working on the problem involved using the distributive property, as indicated in the interview about the solution steps. The subject stated, "After obtaining the result, namely $2x - 8$, then both sides of the equation will become $2x$*

DOI: <https://doi.org/10.24127/ajpm.v13i3.9457>

- $15 = 2x - 8$, where the value of $2x$ on the right side will be moved to the left side, and the value of 15 on the left side will be moved to the right side to collect equivalent values, namely values that have variables and constant values." The procedural steps taken by subject YRH did not align with the concept of the distributive property to combine like terms, which should be operable if these terms are on the same side. The appropriate steps involve using the equality property. In this situation, the student provided a correct answer, but there was a construction process of the distributive property concept in the procedural steps that needed more accuracy. The construction error in this situation is called hole construction. Several other researchers have said that hole construction is an error that students make when they are building their ideas because their thinking is not fully formed during the conceptual construction process (Anggraini et al., 2018; Ni & Halim Fathani, 2018; Subanji, 2015; Wibawa et al., 2018; Wulandari et al., 2021). Therefore, it can be concluded

that subject YRH made a hole construction error in transformational activities based on incomplete rule-based procedures to solve cognitive conflict problems. Apart from the construction hole construction errors, this research also had mis-analogical construction. Here, the AWH subject in the transformational activity of resolving cognitive conflict problems experienced mis-analogical construction errors.

Descriptions of the codes in several of the Figure 6 can be seen in Figure 7. Figure 6 shows that subject AWH changed the steps of the process instead of following the rules for solving linear equations with one variable when dealing with cognitive conflict issues in transformational activities. Instead, in this situation, subject AWH worked on it using quadratic equations. There was an error in the transformational activity performed by subject AWH that did not adhere to the rules. Ideally, the subject responded with steps equivalent to the equation using the distributive and equality properties.

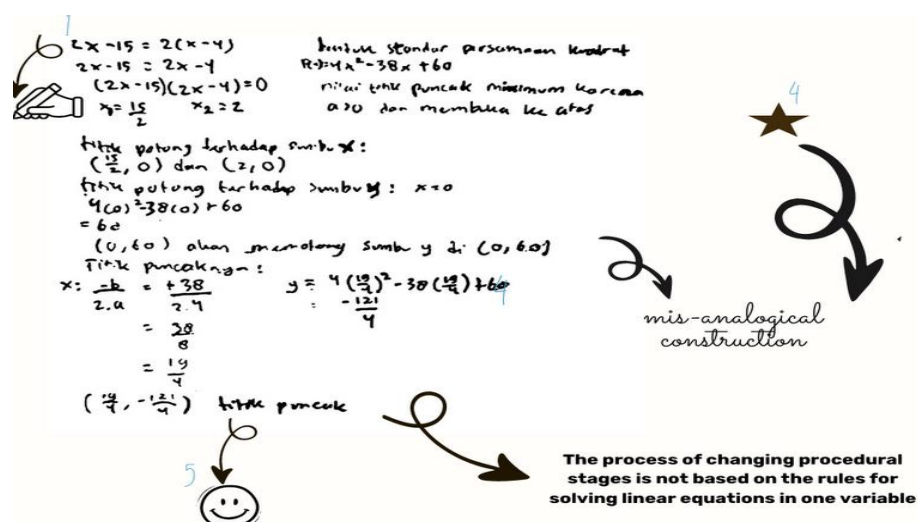
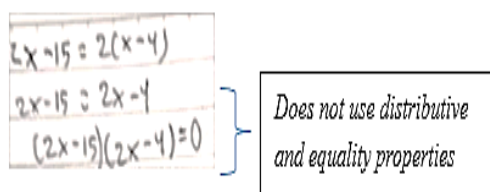


Figure 6. Construction Errors in Transformational Activities of AWH Subjects to Resolve Cognitive Conflict Problems



Figure 7. Codes from transformational activities



Does not use distributive and equality properties

Figure 8. Early errors in the transformational stages made by AWH

During the interview, subject AWH addressed the given problem with the assumption that it was a quadratic equation. The subject stated, "*Determine the intersection point on the x-axis by factoring, determine the intersection point on the y-axis by substituting $x=0$, and determine its vertex with $(-b)/2a$ to find x and then substitute it into the equation.*" Based on the answers and interview results from subject AWH, the student provided an incorrect answer because they equated the concept of a quadratic equation with a linear equation. In this situation, the student experienced a construction error in the form of misanalogical construction. Several researchers agree with this interpretation of the construction error. They say mis-analogical construction is a conceptual construction error when students equate one idea with another (Subanji, 2015; Wulandari et al., 2021). Therefore, it can be concluded that

subject AWH made a mis-analogical construction error in transformational activities based on incomplete rule-based reasoning to solve cognitive conflict problems.

Based on the research findings from the four research subjects, construction errors in transformational activities to solve cognitive conflict problems can be seen in Table 1. This study found four construction errors in problem-solving that may lead to cognitive conflict. Meanwhile, another researcher, Setiawan (2023), identified construction errors in pseudo-thinking in the misplacement of concepts and incorrect misanalogy in addition to operations involving square roots. Wyrasti et al. (2018) looked at misanalogous constructions in set theory, fractions, and sequences for students who messed up when they tried to connect problems with different ways of representing ideas. Both this researcher and others found research results on construction errors in problem-solving that may lead to cognitive conflict. However, what distinguishes this researcher's findings from others is the type of construction error and the mathematical aspects studied. This researcher investigated transformational activities on linear equations with one variable.

DOI: <https://doi.org/10.24127/ajpm.v13i3.9457>

Table 1. Construction Errors in Transformational Activities to Resolve Cognitive Conflict Problems from Research Subjects

Subject research	Construction Errors in Transformational Activities to Resolve Cognitive Conflict Problems	Description
Ak	Experiencing pseudo-construction "correct" errors in incomplete rule-based transformational activities to solve problems that may lead to cognitive conflict.	It occurs when a student provides a correct answer to a problem, but upon further examination, it is found that the student needs to be more accurate in clarifying the answer during transformational activities without determining the solution to solve cognitive conflict problems.
Sar	Experiencing pseudo-construction "incorrect" errors in incomplete rule-based transformational activities to solve problems that may lead to cognitive conflict.	It occurs when a student provides an incorrect answer to a problem, but upon further examination, the student has the correct reasoning and can provide the right answer during transformational activities without simplifying to solve cognitive conflict problems.
YRH	Experiencing hole construction errors in incomplete rule-based transformational activities to solve problems that may lead to cognitive conflict.	It occurs when there is a discrepancy in the construction process of concepts in the student's mind during transformational activities without using the equality property to solve cognitive conflict problems.
AWH	Experiencing mis-analogical construction errors in incomplete rule-based transformational activities to solve problems that may lead to cognitive conflict.	It occurs when a student needs to correct their analogizing a problem with representations of other concepts during transformational activities without using distributive and equality properties to solve cognitive conflict problems.

Subjects Ak, Sar, YRH, and AWH experienced construction errors in transformational activities to solve cognitive conflict problems, placing them in situations that led to cognitive conflict. In these situations, inappropriate procedures were found and categorized as violating a rule (Gal,

2019). The data analysis identified construction errors where subjects Ak, Sar, and YRH provided incorrect steps for equivalent equations, misusing the distributive property in grouping-like terms. Additionally, subject AWH did *not apply the distributive and equality properties*.

DOI: <https://doi.org/10.24127/ajpm.v13i3.9457>

Conceptual construction errors in the form of misanalogical construction lead to cognitive conflict, a viewpoint supported by several researchers. Setiawan (Setiawan et al., 2023) positions conceptual errors as misanalogous when the analogy between problems and applied theory is incorrect and inappropriate in root addition operations. Wyrasti, Sa'Dijah, et al. (2018) looked into misanalogous constructions in set theory, fractions, and sequences for students who messed up when they tried to connect problems with different ways of representing ideas.

Errors made by students in constructing mathematical concepts involve pseudo-thinking and need attention. This aligns with the statement by Anggraini et al. (Anggraini et al., 2018) that students' thinking processes are memorized and spontaneous, not controlling what they think or do and vaguely recalling the procedures. Consequently, students' mathematical concepts must be completed, leading to construction holes. Students' errors in learning algebra, especially in construction, need attention. If promptly addressed, these errors will ensure students' understanding of subsequent algebraic concepts. Therefore, further action must be taken to rectify the emerging conceptual construction errors. However, before taking further action, it is necessary to identify the conceptual construction errors that arise in students. According to other researchers, identifying these construction errors necessitates knowledge of the sources of errors for effective correction. (Anggraini et al., 2018).

CONSLUSIONS AND SUGESSTIONS

Based on the results presented, it can be concluded that there are four

forms of construction errors made by students: "correct" pseudo construction, "incorrect" pseudo construction, hole construction, and mis-analogical construction in transformational activities based on incomplete rule-based approaches to solving problems that may lead to cognitive conflict. The "correct" pseudo-construction error occurs when students provide a correct answer to a problem. Still, upon closer examination, it is revealed that the student is errors in clarifying the answer. The "incorrect" pseudo-construction error happens when students provide an incorrect answer to a problem. Still, upon closer examination, the student has the correct thinking process and can provide the right answer. Hole construction errors occur when there is a mismatch in the construction process of the concept in the student's mind. Mis-analogical construction errors occur when students need to correct their analogizing problems with representations of other concepts.

All four construction errors occur in transformational activities based on incomplete rule-based approaches to solving problems that may lead to cognitive conflict. Therefore, further action must be taken promptly to rectify the arising conceptual construction errors and prevent them from impacting students' understanding of subsequent mathematical concepts. Examining these construction errors is a foundation for educators to improve students' transformational thinking activities in line with solving linear equations with one variable. The findings of this study will offer an opportunity for further research on construction errors in other algebraic thinking activities, such as generational and meta-global-level errors. This will be essential for a more in-depth analysis of students'

DOI: <https://doi.org/10.24127/ajpm.v13i3.9457>

construction errors in algebraic thinking activities when solving problems that may lead to cognitive conflict.

REFERENCES

- Anggraini, D., Kusmayadi, T. A., & Pramudya, I. (2018). Construction of the mathematical concept of pseudo thinking students. *Journal of Physics: Conference Series*, 1022(1), 012010. <https://doi.org/10.1088/1742-6596/1022/1/012010>
- Badawi, A., Richmad, & Agoestanto, A. (2016). Developing Algebra - a case study of the first lessons from the beginning of year 7. *Unnes Journal of Mathematics Education (UJME)*, 5(3), 182–189. <https://doi.org/10.15294/ujme.v5i3.13100>
- Booker, G., & Windsor, W. (2010). Developing Algebraic Thinking: using problem-solving to build from number and geometry in the primary school to the ideas that underpin algebra in high school and beyond. *Procedia - Social and Behavioral Sciences*, 8, 411–419. <https://doi.org/10.1016/j.sbspro.2010.12.057>
- Coles, A., & Brown, L. (1998). Developing Algebra - a case study of the first lessons from the beginning of year 7. *Proceedings of the British Society for Research into Learning Mathematics*, 18(November), 17–22. <https://citeseerx.ist.psu.edu/document?repid=rep1&type=pdf&doi=eb308dce5acaa6760962d2b34ec29f2430b0e81e>
- Creswell, J. W. (2014). *Research design: qualitative, quantitative, and mixed methods approaches* (4th (ed.)). SAGE Publications, Inc.
- Creswell, J. W., & Creswell, J. D. (2018). *Research Design Qualitative, Quantitative, and Mixed Methods Approaches* (Fifth Edit). SAGE Publications Inc.
- Fraser, D. (2007). Using cognitive conflict to promote a structural understanding of grade 11 algebra. *ProQuest Dissertations and Theses*, 174.
- Fujii, T. (1987). The role of cognitive conflict in understanding mathematics. In Begeron, J., Herscovics, N., Kieran, C. (Eds.) *Proceedings of the Eleventh International Conference for the Psychology of Mathematics Education. Volume III*, 141–147.
- Fujii, T. (2003). Probing students' understanding of Variables through cognitive conflict problems? Is the concept of a variable so difficult for students to understand? In Neil A. Pateman, B. J. Dougherty, & J. T. Zilliox (Eds.), *Proceedings of the 27th Conference of the International Group for the Psychology of Mathematics Education held jointly with the 25th Conference of PME-NA* (pp. 1–65). CRDG, College of Education, University of Hawai'i.
- Gal, H. (2019). When the use of cognitive conflict is ineffective—problematic learning situations in geometry. *Educational Studies in Mathematics*, 102(2), 239–256. <https://doi.org/10.1007/s10649-019-09904-8>
- Halimah, Subanji, & Septi Nur Afifah, D. (2019). Student's cognitive conflict form problem solving on mathematics. *Journal of Physics: Conference Series*, 1339(1), 012127. <https://doi.org/10.1088/1742-6596/1339/1/012127>

DOI: <https://doi.org/10.24127/ajpm.v13i3.9457>

- Imam, N., & Kusmaryono, I. (2023). Transforming Students' Pseudo-Thinking Into Real Thinking in Mathematical Problem Solving. *International Journal of Educational Methodology*, 9(3), 477–491.
<https://doi.org/10.12973/ijem.9.3.477>
- Inganah, S., Nabila, A. I., & Putri, O. R. U. (2021). Kesalahan Konstruksi Konsep Matematis Dalam Proses Representasi Visual Mahasiswa. *AKSIOMA: Jurnal Program Studi Pendidikan Matematika*, 10(3), 1776.
<https://doi.org/10.24127/ajpm.v10i3.3993>
- Irawati, Zubainur, C. M., & Ali, R. M. (2018). Cognitive conflict strategy to minimize students' misconception on the topic of addition of algebraic expression. *Journal of Physics: Conference Series*, 1088, 012084.
<https://doi.org/10.1088/1742-6596/1088/1/012084>
- Kieran, C. (2004). Algebraic Thinking in the Early Grades: What Is It? *Mathematics Educator*, 8(1), 139–151.
- Leron, U., & Hazzan, O. (2009). Intuitive vs analytical thinking: four perspectives. *Educational Studies in Mathematics*, 71(3), 263–278.
<https://doi.org/10.1007/s10649-008-9175-8>
- Limón, M. (2001). On the cognitive conflict as an instructional strategy for conceptual change: a critical appraisal. *Learning and Instruction*, 11(4–5), 357–380.
[https://doi.org/10.1016/S0959-4752\(00\)00037-2](https://doi.org/10.1016/S0959-4752(00)00037-2)
- Maharani, I. P., & Subanji, S. (2018). Scaffolding Based on Cognitive Conflict in Correcting the Students' Algebra Errors. *International Electronic Journal of Mathematics Education*, 13(2).
<https://doi.org/10.12973/iejme/2697>
- Malloy, C., Price, J., Willard, T., & Sloan, L. L. "Butch." (2002). *Pre-Algebra*. USA TODAY Snapshots; Dinah-Might Activities, Inc.
- Ni, R., & Halim Fathani, A. (2018). Kesalahan Konstruksi Konsep Matematika Dan Scaffolding-Nya Construction Error of Mathematical Concepts and His Scaffolding. *Jurnal Pendidikan Dan Pembelajaran*, 3(2), 162–171.
- Pape, S. J. (2004). Middle School Children's Problem-Solving Behavior: A Cognitive Analysis from a Reading Comprehension Perspective. *Journal for Research in Mathematics Education*, 35(3), 187.
<https://doi.org/10.2307/30034912>
- Pratiwi, E. ;, Nusantara, T. ;, Susiswo, S. ;, Muksar, M. ;, & Subanji, S. (2019). Characteristics of Students' Cognitive Conflict in Solving a Problem Based on Information Processing Theory. *International Journal of Learning, Teaching and Educational Research*, 18(2), 76–88.
<https://doi.org/10.26803/ijlter.18.2.6>
- Pratiwi, E., Nusantara, T., Susiswo, S., & Muksar, M. (2022). Routines' errors when solving mathematics problems cause cognitive conflict. *International Journal of Evaluation and Research in Education (IJERE)*, 11(2), 773.
<https://doi.org/10.11591/ijere.v11i2.21911>
- Putri, R. W., & Indrawatiningsih, N. (2023). Defragmenting Student

DOI: <https://doi.org/10.24127/ajpm.v13i3.9457>

- Construction Holes in Solving System of Absolute Value Equations. *Jurnal Riset Pendidikan Dan Inovasi Pembelajaran Matematika*, 6(2), 131–142. <https://doi.org/10.26740/jrpiipm.v6n2.p131-142>
- Sari, A. N. (2021). The Profile of Cognitive Conflict with Intervention in Understanding Algebra of Students at Pangkep State Polytechnic of Agriculture. *HISTOGRAM: Jurnal Pendidikan Matematika*, 4(2), 545–564. <https://doi.org/10.31100/histogram.v4i2.772>
- Sela, H. (2008). Coping With Mathematical Contradictions With Peers. *Paper Presented at Topic Study Group 18, ICME 11, July*, 1–9.
- Sela, H., & Zaslavsky, O. (2007). Resolving Cognitive Conflict With Peers – Is There a Difference Between Two and Four? *Proceedings of the 31 Conference of the International Group for the Psychology of Mathematics Education*, 4, 169–176.
- Setiawan, I., Purwanto, P., Sukoriyanto, S., & Nengah, I. (2023). Cognitive Conflict Based on Thinking Errors in Constructing Mathematical Concept. *International Journal of Educational Methodology*, 9(4), 631–643. <https://doi.org/10.12973/ijem.9.4.631>
- Subanji. (2015). *Teori Kesalahan Konstruksi Konsep dan Pemecahan Masalah Matematika* (T. Nusantara (ed.); I). Universitas Negeri Malang (UM Press).
- Subanji, & Nusantara, T. (2013). Karakterisasi Kesalahan Berpikir Siswa Dalam Mengonstruksi Konsep Matematika. *Jurnal Ilmu Pendidikan*, 19(2), 208–2017. <https://journal.um.ac.id/index.php/jip/article/view/4215/1201>
- Sutopo. (2014). Counterexample in Cognitive Conflict As Factor Influencing Conceptual Change. *QIJIS (Qudus International Journal of Islamic Studies)*, 2(2), 198–218. <http://dx.doi.org/10.21043/qijis.v2i2.1567>
- Tirosh, D., & Graeber, A. O. (1990). Evoking Cognitive Conflict to Explore Preservice Teachers' Thinking about Division. *Journal for Research in Mathematics Education*, 21(2), 98–108. <https://doi.org/10.5951/jresmetheduc.21.2.0098>
- Vinner, S. (1997). The pseudo-conceptual and the pseudo-analytical thought processes in mathematics learning. *Educational Studies in Mathematics*, 34(2), 97–129. <https://doi.org/10.1023/A:1002998529016>
- Walida, S. El, Sa'dijah, C., Subanji, & Sisworo. (2022). A Portrait of Controversial Mathematics Problems and Students' Metacognitive Awareness: A Case of Indonesia. *Journal of Higher Education Theory and Practice*, 22(12), 51–62. <https://doi.org/10.33423/jhetp.v22i12.5462>
- Wibawa, K. A., Nusantara, T., Subanji, & Nengah Parta, I. (2018). Defragmentation of Student's Thinking Structures in Solving Mathematical Problems based on CRA Framework. *Journal of Physics: Conference Series*, 1028(1), 012150. <https://doi.org/10.1088/1742-6596/1028/1/012150>

DOI: <https://doi.org/10.24127/ajpm.v13i3.9457>

- Wulandari, S., Kamid, K., & Haryanto, H. (2021). Analisis Kesalahan Kontruksi Konsep Pada Materi Bangun Ruang Sisi Datar Berdasarkan Pemberian Scaffolding. *AKSIOMA: Jurnal Program Studi Pendidikan Matematika*, 10(4), 2801. <https://doi.org/10.24127/ajpm.v10i4.4151>
- Wyrasti, A. ., Sa'dijah, C., As'Ari, A. R., & Sulandra, I. M. (2018). The Misanalogical Construction of Undergraduate Students in Solving Cognitive Conflict Identification Task. *International Electronic Journal of Mathematics Education*, 14(1). <https://doi.org/10.12973/iejme/3961>
- Wyrasti, A. F., Sa'dijah, C., As'ari, A. R., & Sulandra, I. M. (2018). The Misanalogical Construction of Undergraduate Students in Solving Cognitive Conflict Identification Task. *International Electronic Journal of Mathematics Education*, 14(1), 33–47. <https://doi.org/10.12973/iejme/3961>
- Zazkis, R., & Chernoff, E. J. (2008). What makes a counterexample exemplary? *Educational Studies in Mathematics*, 68(3), 195–208. <https://doi.org/10.1007/s10649-007-9110-4>