MATHEMATICAL CONNECTION OF GEOMETRY PROBLEMS BASED ON NUMERACY LITERACY THROUGH THE POLYA FRAMEWORK

Ajeng Gelora Mastuti1, Syafruddin Kaliky2, Aliati Siolimbona3, Jiran Rano4

1,2,3,4 Institut agama Islam Negeri Ambon, Ambon, Indonesia

E-mail: ajeng.gelora.mastuti@iainambon.ac.id
kaliykysyafruddin@iainambon.ac.id
aliatiisolombona@gmail.com
jiranrano12@gmail.com

Received 09 August 2022; Received in revised form 01 February 2023; Accepted 15 February 2023

Abstract

The results of the 2018 Program for International Student Assessment (PISA) study released by The Organization for Economic Cooperation and Development (OECD) show that the numeracy literacy skills of Indonesian students are at a low level, namely getting a score of 379 out of the OECD average score of 487 (OECD, 2019). As a result, students tend not to be able to solve problems correctly, and the cause is a need for mathematical connection skills. This study aims to describe the mathematical connections based on numeracy literacy in geometry. This research is a qualitative descriptive study. The participants in this study were 38 grade 7 students from 3 different Madrasas in Maluku. Two students will take the research subjects from saturated data after being given a numeracy literacy-based test. In this study, students were asked to state the results of their thoughts on solving geometry concept problems given orally. Analyzing the research data was carried out through three stages: data reduction, data presentation, and drawing conclusions. The findings in this study of mathematical connections within the Polya framework can be divided into two: structured analytics and unstructured analytics.

Keywords: Mathematical connections, numerical literacy, geometry problem solving
INTRODUCTION

The 2012 and 2015 PISA results still place Indonesian students at the bottom. Indonesia's position still needs to be improved to Vietnam, Thailand and Malaysia, which are in the same region as Southeast Asia (Fenanlampir et al., 2019). In Indonesia, numeracy literacy is still in the low category. This is based on the results of the 2018 Program for International Student Assessment (PISA) study released by The Organization for Economic Cooperation and Development (OECD), which shows that the numeracy literacy skills of Indonesian students are at a low level, namely getting a score of 379 out of an average score. OECD 487 (OECD, 2019). Students are not able to solve problems correctly in geometry problems.

So far, students view learning geometry as a unit of learning mathematics that is classified as difficult; Among other things, it can be seen that students still experience difficulties in recognizing and understanding geometric shapes and elements (Alghadari et al., 2020). Furthermore, students' difficulties in learning geometry are closely related to their ability to solve mathematical problems (Rochmad et al., 2021). Therefore, learning geometry must be adapted to students' mathematical abilities, including the level of students' thinking in geometry and students' mathematical connections (Huffman et al., 2017). Thinking skills can also be defined as a cognitive process broken down into concrete steps and used as a guide for thinking (Astra et al., 2021).

Based on the results of observations on Madrasah students in Maluku, it was found that the ability to make mathematical connections in numeracy literacy problems for geometry was still low. This is because students rarely get experience with literacy questions. Mathematical connection ability is one of the five abilities that students must have in solving mathematical problems, including 1) problem-solving ability, 2) representational ability, 3) communication ability, 4) reasoning ability, and 5) mathematical connection ability. According to NCTM, one of the basic math skills needed is the ability to connect mathematically.

Mathematical connection skills are important (Ayunani et al., 2020; Rohid et al., 2019; Siregar & Daut Siagian, 2019). The NCTM mentions the importance of mathematical connections for students; that is, it is used to help students broaden their perspectives, to see mathematics as a unified whole rather than as a series of separate topics and recognize its relevance and usefullness both inside and outside school (Mastuti, Abdillah, & Rijal, 2022). Students' understanding will be deeper if students can relate concepts that are already known to students with new concepts that students will learn (Lodge et al., 2018). Thus, with mathematical connections, students will feel the benefits of learning mathematics, and the attachment of students' understanding of the concepts they are learning will last longer (Bringula et al., 2021). This is in line with the observations made by giving preliminary test questions to several junior high school students in Maluku to be completed. It was found that students can represent problems into geometric concepts, especially flat shapes, and solve the problem to get a solution. Some students can do mathematical problem solving well and show good mathematical connections. However, some still make conceptual
errors and have not shown a complete mathematical connection at the problem-solving stage.

Mathematical connections can influence reasoning in numeracy literacy tasks (Hwang & Ham, 2021). According to the 2018 PISA framework, numeracy literacy “describes the knowledge content processes, and contexts that are reflected in mathematics assessment questions” and shows how students perform in mathematics (OECD, 2019). The construct of mathematical literacy describes individual competencies to reason mathematically and use mathematical concepts, procedures, facts, and tools to describe, explain, and predict phenomena. This conceptualization of mathematical literacy supports the importance of engaging students in pure mathematics tasks (mathematical reasoning) and their exploration of the abstract world of mathematics (using mathematical concepts, procedures, facts, and tools) (Hwang & Ham, 2021).

Based on previous studies on mathematical connections and learning geometry, we need to look at other things that are different. Therefore, this study will look at the structure of thinking based on different problem-solving using the Polya framework. This thinking structure will be the findings of our research.

Therefore, this study aims to describe mathematical connections based on numeracy literacy in geometry. Furthermore, we conceptualize assignments based on higher-order thinking skills, a feature of the Indonesian Madrasah Competency Assessment. Therefore, the primary key of this research lies in the students’ mathematical connections based on the structure of their thinking.

**METHOD**

This research is a qualitative descriptive study. Qualitative research explains the mathematical connection to the problem of flat shapes based on a numeracy literacy test based on the Polya framework. This study also aims to provide a means for students to realize mathematical connections through problems that are deliberately designed by researchers so that researchers themselves are the main tool for systematically analyzing methods.

The participants in this study were 38 and 7th-grade students from 3 different Madrasahs in Maluku. The research subjects will be taken from two students from saturated data after students are given a numeracy literacy-based test. The selected subjects are part of the students who have high numeracy literacy scores based on the Asesmen Kompetensi Madrasah Indonesia. The research will be held at the end of the odd semester of 2022 at MTs Negeri Ambon in Kebun Cloves and Air Salobar, and MTs Al Anshor. Only students with high competence were taken to determine the students’ mathematical connections in a structured manner. Data was collected through video recordings and interviews with students. Data analysis is centred on each student's actions when working on the concept of flat shapes based on numeracy literacy. We analyzed each piece of data recorded on the video. Table 1 shows the scores for the numeracy literacy category, which are used to obtain scores on the first test.

Table 1. Category of numerical ability

<table>
<thead>
<tr>
<th>Score</th>
<th>Category</th>
</tr>
</thead>
<tbody>
<tr>
<td>16 ≤ n ≤ 24</td>
<td>High</td>
</tr>
<tr>
<td>8 ≤ n &lt; 16</td>
<td>Moderate</td>
</tr>
<tr>
<td>0 ≤ n &lt; 8</td>
<td>Low</td>
</tr>
</tbody>
</table>

| DOI: https://doi.org/10.24127/ajpm.v12i1.5924 |
The research instrument is a test of understanding the concept of Geometry to see mathematical connections based on numeracy literacy in Asesmen Kompetensi Madrasah Indonesia. Mathematical connection abilities stated in the objectives of school mathematics learning are: to understand mathematical concepts to clarify relationships between concepts, and to apply concepts or logarithms flexibly, effectively and precisely to solve problems (Baiduri et al., 2020).

The test instrument is in the form of a flat wake problem as below:

“A photo measuring 4 cm × 6 cm will be enlarged so that the photo's width is 12 cm. What is the minimum photo paper size needed to make the enlarged photo?”

The flat wake problem test was analyzed qualitatively using mathematical connection indicators based on the Polya framework, as shown in Table 2.

Table 2. Indicators of mathematical connections based on the Polya framework

<table>
<thead>
<tr>
<th>Mathematical connection</th>
<th>Polya's Framework</th>
<th>Mathematical Connection Process Indicator</th>
</tr>
</thead>
<tbody>
<tr>
<td>Connection in mathematics</td>
<td>Understand the Problem</td>
<td>➢ Writing mathematical facts that are known to the given problem&lt;br&gt;➢ Writing down what was asked&lt;br&gt;➢ Identifying mathematical concepts from information on everyday life problems</td>
</tr>
<tr>
<td></td>
<td>Planning a Problem Resolution</td>
<td>➢ Finding the relationship that is asked with facts, concepts, and mathematical principles on the problem&lt;br&gt;➢ Expressing mathematical procedures or operations that will be used to solve problems</td>
</tr>
<tr>
<td>Mathematical connection with other sciences</td>
<td>Carry out Problem Solving</td>
<td>➢ Using connection to the planned principles&lt;br&gt;➢ Applying mathematical concepts in problem solving&lt;br&gt;➢ Writing with a regular pattern in solving problems&lt;br&gt;➢ Using procedures according to the planned strategy</td>
</tr>
<tr>
<td>Mathematical connections in everyday life</td>
<td>Re-check</td>
<td>➢ Checking facts, principles, and procedures used in solving problems&lt;br&gt;➢ Checking whether the steps used are correct&lt;br&gt;➢ Checking the calculation obtained</td>
</tr>
</tbody>
</table>

In this study, students were asked to state the results of their thoughts in solving the problem of the concept of geometry given orally. After working on the problem, the researcher interviewed the subject to strengthen the research results. Data analysis is data that is compiled, revised, and choreographed (Creswell, 2013). The data analysis technique of the research results was carried out through three stages: data reduction, data presentation, and drawing conclusions. In data reduction, the researcher selects important points from the data according to predetermined indicators.
Then the researcher presented descriptive data that was arranged systematically. Meanwhile, the conclusion is obtained by the researcher taking data from the research evidence.

RESULT AND DISCUSSION

Of the 38 participants involved, 30 students had high scores based on the arithmetic literacy-based flat wake test results. The data acquisition of student scores can be seen in Table 3. Of the 25 students who scored high after being given the first test, two students were consistent after being given the second test. Therefore, the second test is given based on indicators of conceptual understanding. Table 3 shows the results of both Subject 1 (S1) and Subject 2 (S2) tests based on the mathematical connection rubric.

Table 3. Results of the mathematical connection scoring rubric

<table>
<thead>
<tr>
<th>Name</th>
<th>Indicator</th>
<th>Score</th>
</tr>
</thead>
<tbody>
<tr>
<td>S1</td>
<td>Looking for relationships between various representations of concepts and procedures</td>
<td>3</td>
</tr>
<tr>
<td></td>
<td>Understanding the relationship between mathematics topics</td>
<td>3</td>
</tr>
<tr>
<td></td>
<td>Understand the equivalent representation of the same concept</td>
<td>3</td>
</tr>
<tr>
<td></td>
<td>Using mathematics in other fields of study or in daily life</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td>Finding the connection of one procedure to another with its equivalent representation</td>
<td>3</td>
</tr>
<tr>
<td>Total</td>
<td></td>
<td>14</td>
</tr>
<tr>
<td>S2</td>
<td>Looking for relationships between various representations of concepts and procedures</td>
<td>3</td>
</tr>
<tr>
<td></td>
<td>Understanding the relationship between mathematics topics</td>
<td>3</td>
</tr>
<tr>
<td></td>
<td>Understand the equivalent representation of the same concept</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td>Using mathematics in other fields of study or in daily life</td>
<td>3</td>
</tr>
<tr>
<td></td>
<td>Finding the connection of one procedure to another with its equivalent representation</td>
<td>3</td>
</tr>
<tr>
<td>Total</td>
<td></td>
<td>14</td>
</tr>
</tbody>
</table>

The Mathematical Connection Process of S1

Understand the Problem

S1, before completing the questions given, first read the questions given. After reading, S1 seemed to start solving by first simplifying the problem into a picture and then writing down the information contained in the problem in a known form. It looks like Figure 1.

Based on Figure 1 part A, it appears that S1, in problem-solving, tends to make an image before being enlarged, measuring 4 cm × 6 cm, and in picture 1 part B, where the subject makes an image after being enlarged, measuring 12 cm × p cm. Two rectangular images where the second image is the result of the enlargement of the first image. In addition, S1 also writes down the known size of the photo.
Based on Figure 1 part A, it appears that the subject S1, in problem-solving, tends to make an image before it is enlarged, measuring 4 cm × 6 cm and in picture 1 part B, where the subject makes an image after being enlarged measuring 12 cm × p cm. Two rectangular images where the second image is the result of the enlargement of the first image. In addition, S1 also writes down the known size of the photo. Based on the S1 statement: "The size of the photo before it was enlarged was 4 cm x 6 cm then it was enlarged to 12 cm". This shows that AS subjects can write down known mathematical facts and see the relationship between various representations of concepts and procedures.

S1 can also describe a rectangular photo frame because it can make it easier to solve the problem. This shows that S1 can identify mathematical concepts from information about everyday life problems and see the relationship between mathematical topics.

Planning a Problem Resolution

After revealing all the information in the problem, S1 plans the completion stage by writing a formula or formula. This looks like Figure 2.

Figure 2 shows that S1 makes an example of \( l_0, l_1, p_0, \) and \( p_1 \) as symbols of the initial and final length and width. In addition, the S1 also wrote the formula completely. S1 devises a plan to create the equation \( \frac{l_0}{p_0} = \frac{l_1}{p_1} \). This shows that S1 connects mathematical concepts by connecting the concepts of comparison and similarity. S1 makes
mathematical connections in everyday life when expressing a plan to find the minimum photo paper size. When planning to find $p_1$ to determine the minimum photo paper size, it can also be said to have a mathematical connection with other sciences. Because applying the use of minimal materials for profit is also used in economics and also the science of measurement used by architects.

Carry out Problem Resolution

After S1 writes a formula, the subject continues his work by looking for the final result of solving the problem. It can be seen that the results of the work on the S1 questions look like Figure 3. Based on Figure 3, it can be seen that S1 also show that in the completion process, students first substitute the known values into the comparison formula. Then using algebraic properties followed by multiplication, division and addition operations (procedure stage), students conclude the final answer correctly from these results.

![Figure 3. Carry out Problem Resolution of S1](image)

Based on Figure 3 and interviews, S1 has made a connection between mathematical concepts, namely by connecting the length and width of the original photo size with the results of the enlarged photo length and width with the concept of worth comparison. Mathematical relationships with measurements in a field such as the science of architects appear when S1 compares length and width. When S1 compares the length and width, it means that S1 has connected the problems of everyday life with mathematical concepts.

Re-checking

The S1 process was re-checked by checking the operation and strategy calculation results. This means that S1 makes connections between mathematical concepts by re-examining the results obtained by connecting the concept of comparison with similarity. This is obtained from the quote during the S1 interview as follows.

“...I'm sure of my answer after several checks. I have determined the minimum photo size that can fit into the enlarged frame”.

Overall, the process of mathematical connection S1 in solving problems by making connections in mathematics, mathematical connections with other sciences, and mathematical connections in everyday life. At the stage of understanding the problem of the connection process in mathematics, compiling a mathematical model in the form of an equivalent comparison. At the solution planning stage, the connection process in mathematics is done by planning a solution (choosing a comparison method to apply the concept of congruence). Completing a
solution with equivalent ratios to determine the minimum photo size is a connection process in mathematics at the problem-solving stage. At the stage of re-checking the connection process in mathematics by reviewing the calculations and strategies used in solving problems (Baiduri et al., 2020). The process of connecting mathematics with other sciences (economics and measurement science in architecture) when S1 connects the meaning of minimal size at understanding a problem, planning a solution, and implementing a solution.

The Mathematical Connection Process of S2

Understand the Problem

The S1 process in understanding the problem does not appear on the written answer sheet. S2 does not write down any information about the problem, starting from presenting known and asked facts. This is because the information presented in the problem is very clear. Instead, S2 understands the problem by mentioning the important elements of the problem, namely things that are known and asked. From the interview, S2 knew that the photo was a rectangle because it was easier and faster to solve the problem. This shows that S2 can see the relationship between mathematics topics.

Planning a Problem Resolution

Furthermore, S2 writes down the information in the problem and then S2 plans the completion stage by rewriting the formula and simulating the size of the enlarged photo in the form of an image. Next, S2 writes down each size belonging to each of the rectangles depicted. This looks like the following Figure 4.

Figure 4. Planning a Problem Resolution of S2

Based on Figure 4, S2, in solving the problem, tends to make an image before it is enlarged to 4 cm × 6 cm and makes an image after it is enlarged to 12 cm × P2 cm. Next, S2 writes the formula with symbols l1, l2, P1, and P2 as symbols of the initial and final length and width. This is reinforced by the results of interviews that have been carried out with S2 where the subject said as his expression, namely "l1 is the original area before being enlarged, l2 is wide after it is enlarged, P1 is the initial length before it is enlarged and P2 is long after it is enlarged".

Based on the results in Figure 4 and the interview, it is obtained that S2 can understand the problem given. S2 makes a formula or formula well by first making mathematical symbols of the problem to facilitate the completion process. More specifically, S2 reveals that the images made have the same shape but different sizes (similarity). This is referred to as the comparison concept used in solving problems. This shows that S2 can see the relationship between several concepts, namely the concept of rectangle, similarity and comparison.

Then to be able to complete or apply mathematics in concepts or other fields of study, S2 also argues that the problem is also related to other fields of study, one of which is physics subjects which are usually related to the comparison equation between mass and acceleration based on the force used. This is reinforced by the results of the interview with S2: "From my solution, I think the comparison formula that I use
is the same as when I work on physics problems that I have worked on, I also use the comparison formula between mass and acceleration with the same force”.

**Carry out Problem Resolution**

After S2 writes the formula or formula into the answer sheet, then S2 continues the results of his work by looking for the final result of solving the problem. It can be seen that the results of the S2 problem solving work look like the Figure 5.

![Figure 5. Carry out Problem Resolution of S2](image)

Based on the results of S2 work in Figure 5, it can be seen that S2 can solve the problem by obtaining the final result with the correct answer. Based on Figure 5, it can be seen that S2 also shows that in the completion process, students first substitute the values that have been known into the comparison formula and then use algebraic properties followed by the multiplication division and addition (procedure stage). From the results then, the students conclude the final answer correctly.

**Re-checking**

Based on interview excerpts, the S2 process checks the results obtained by checking the calculations and ensuring the results obtained are minimal photo materials. The mathematical connections made by S2 in re-checking are 1) mathematical connections by checking comparison operations and rectangular area, 2) mathematical relationships with other sciences (physics), namely determining the minimum size of photos using the same comparison as when the subject makes comparisons between masses. Acceleration and force. The process of mathematical connection S2 in solving problems is done by connecting mathematical concepts and mathematical relationships with other sciences in everyday life. Connections in mathematics are made at each stage of Polya. The mathematical connection process is carried out by subjects P1 and L1 in solving problems by making mathematical connections in mathematics, mathematical connections with other sciences, and mathematical connections in everyday life.

Both subjects have almost the same mathematical connection in solving geometric problems. The difference is if the S1 understanding of the problem is more structured and analytic, while the S2 is unstructured. This agrees with Firdaus et al. (2019), that students who think analytical structures tend to solve problems very orderly manner, paying attention to Step by Step in detail. Meanwhile, students who understand unstructured analytical problems tend to imagine more than write and choose to use faster problem solving (Abdillah & Mastuti, 2018; Mastuti & Prayitno, 2023). Mathematical connection ability is important because it has the same nature as systematic and structured science that contains interrelated concepts (Mainali, 2021). Problem-solving carried out by the two subjects is solving everyday
problems, making it easier for students to arrange and solve problems by linking each concept. This is in line with Mastuti et al. (2022), mathematical connections also help students to see the relationship between mathematics and everyday life. Mathematical connections can make students understand a concept and help students improve their understanding of mathematical concepts. In addition, mathematical connections help students provide mathematical models that connect concepts, data, and situations (Mastuti, Abdillah, & Rijal, 2022; Siregar & Daut Siagian, 2019).

CONCLUSION AND RECOMMENDATION

Mathematical connection ability is important because it has the same nature as systematic and structured science that contains related concepts. Students’ mathematical connections can be observed in solving geometric problems through the Polya framework. Based on the results and discussion findings, mathematical connections in the Polya framework can be divided into two, namely structured analytics and unstructured analytics. First is the connection of mathematics with structured analytics, where thinking students tend to solve problems very orderly manner, paying attention to step-by-step detail. Second is the connection of unstructured analytic mathematics, in which students’ problem understanding tends to be more imaginative than writing and chooses to use faster problem-solving.

This research needs to be developed for students with moderate and low numeracy literacy. Furthermore, it becomes a recommendation for teacher learning.

REFERENCES


