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QUALITATIVE THINKING LEVEL FOR GEOMETRY LEARNING BASED ON RIGOROUS MATHEMATICAL THINKING (RMT) APPROACH

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Abstrak

Penelitian ini merupakan penelitian *design research* tipe *validation study* yang bertujuan untuk mengembangkan learning trajectory pada pembelajaran geometri topik geometri tiga dimensi untuk menunjang kemampuan berpikir kualitatif siswa dengan pendekatan Rigorous Mathematical Thinking (RMT). Hypothetical Learning Trajectory (HLT) akan disempurnakan sepanjang pembelajaran yang dilakukan di dalam kelas. Kegiatan dibuat dan dirancang dalam bentuk lembar kerja siswa (LKPD) untuk menunjang proses pembelajaran. Subjek penelitian ini adalah siswa kelas XII SMA Negeri 1 Palembang, sedangkan kelompok fokus penelitian ini adalah 6 siswa dengan kemampuan yang bervariasi yaitu tinggi, sedang, dan rendah yang dipilih berdasarkan hasil prestasi yang didokumentasikan oleh guru dalam semester sebelumnya serta rekomendasi guru mata pelajaran matematika. Ada tiga tahapan dalam melakukan penelitian ini, yaitu: (1) *preparing the experiment*; (2) *design experiment* dibagi menjadi *preliminary experiment* dan *teaching experiment;* (3*) retrospective analysis*. Data penelitian dikumpulkan dan akan dianalisis secara kualitatif. Hasil penelitian menunjukkan bahwa lintasan belajar yang dibuat menggunakan tingkat berpikir kualitatif dengan indikator yang sering terlihat pada siswa kelas XII MIPA 7 SMA Negeri 1 Palembang adalah indikator visualisasi dan pelabelan. Lintasan belajar yang diperoleh dari penelitian ini didasarkan pada LKPD yang telah dilakukan oleh siswa dimana masalah berkontribusi untuk membangun micro teaching dalam pembelajaran dimensi ketiga. (10pt)

Kata kunci: *Design research; three-dimensional geometry; learning trajectory; RMT*. (10pt)

Abstract

This research is a design research type validation study that aims to develop a learning trajectory on the study of geometry topics of three-dimensional geometry to support students' qualitative thinking skills with a Rigorous Mathematical Thinking (RMT) approach. Hypothetical Learning Trajectory (HLT) will be refined throughout the lesson carried out in the classroom. The activities were created and designed in the form of student worksheets (LKPD) to support the learning process. The subjects of the study were ninth grade students of SMA Negeri 1 Palembang, meanwhile, the focus group of this study was 6 students with varied abilities, namely high, medium, and low who were selected based on the achievement result documented by the teacher in the previous semester as well as the recommendations of teachers of mathematics subjects. There are three stages in conducting this research, namely: (1) Preparation Phase; (2) Design experiments divided into pilot experiments and teaching experiments; (3) retrospective analysis. Research data is collected and will be analyzed qualitatively. The results showed that the learning trajectory made using qualitative thinking levels with indicators often seen in students of SMA Negeri 1 Palembang class XII MIPA 7 is an indicator of visualization and labeling. The learning trajectory obtained from this study is based on LKPD that has been done by students where the problem contributes to building micro-teaching in third-dimensional learning.

***Keywords**: Design research; three-dimensional geometry; learning trajectory; RMT*

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INTRODUCTION

In mathematics, geometry is one of the fields that have a logical and abstract structure. At the school level, there are two mathematical standards, one of which is the content standard which contains material taught in schools namely algebra, geometry, measurement, data analysis and probabilistic (National Council of Teachers of Mathematics, 2000). Geometry lessons began to be learned from an early age because they could develop ways of thinking in understanding shapes and spaces as well as shaping students' memory of concrete, and abstract objects, and shaping students' ability to visualize images (Fauzi & Arisetyawan, 2020; Yunita, Maharani, & Sulaiman, 2019). In studying geometry, it takes learning theory. The theory used is van hiele theory.

Van Hiele theory is a theory of learning used in geometry, where in this theory there is a level from level 0 to level 4 according to the level of abstraction possessed by students (Nihayah, 2021; Nuraini, Nur'aeni, & Ganda, 2021). Due to the level of ability that each student has been different, van Hiele's theory is needed to fulfill that ability. Each level of thinking ability of students has certain criteria that can indicate differences in students' understanding of solving geometry problems (Akbar, Handayani, & Mirza, 2020). As a consequence of the differences in students' abilities, problem-solving and spatial skills are also needed because this ability is the basis for maintaining students' arguments logically so that students can understand and solve the problems given (Soraya, Utami, & Nirawati, 2021) one of the materials contained in geometry is three-dimensional matter. This material involves the abilities mentioned above.

However, the learning of three-dimensional geometry is still said to be weak since many students have difficulty mastering the material due to the need for abstract visualization skills (Sumarni & Prayitno, 2016; labulan, 2016). Students also have difficulty at the time of using concepts, principles, and problems given that difficult to understand (Fauzi & Arisetyawan, 2020). Many causes of difficulties experienced by students, including students still unable to use the knowledge they have on geometry and lack of thoroughness in analyzing problems that result in a lack of information when solving the problems (Sholihah & Afriansyah, 2018). Students also often have difficulty in explaining how they found the answer and only write down the answer directly and then draw conclusions also, students tend to be silent when it is difficult to compose their arguments (Shahrill & Clarke, 2014).

Therefore, an approach is needed in learning to help students understand the problems given. One approach that can be used is the Rigorous Mathematical Thinking (RMT) approach. The methods of the RMT approach include steps for students to be able to use cognitive function as a general psychological tool based on the function of its structure to establish a high level of cognitive processes (Sundawan, Irmawan, & Sulaiman, 2019). The RMT approach is effective to use because it can develop high-level mathematical thinking skills and develop other student skills also the RMT can make good interactions during the learning process resulting in students being able to focus on the time learning progresses (Dayat Hidayat,

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Ahmad Wachidul Kohar, Nina Rinda Prihartiwi, Husni Mubarok, & Abebayehu Yohannes, 2021; Fatmawanti1 & Fitriyani, 2019)

The RMT is an approach invented by James T. Kinard. Vygotsky's theory of psychological tool theory and Feuerstein's theory of mediated learning experience (MLE) in which both theories can lead to activity in cognitive function (Yunita et al. 2019; Kinard & Kozulin, 2008). Two major theories have relevance to combine the theory so that students can think starting from a simple thing (Gravemeijer & Van Eerde, 2009). The psychological tools used can be symbols, images, tables, graphs, or others that have a special meaning as a bridge for students in understanding concepts. RMT approach emphasizes interaction and mediation between the teacher and the student, where the situation can make the student understand the material and concepts given well, and come up with the student's ideas (Fitriyani & Khasanah, 2018; Sujalmo, 2013). In the RMT approach, there are activities in cognitive function that are divided into three levels of thinking, namely qualitative thinking level, quantitative thinking level, and abstract relational thinking level (Fuadiah 2017; Kinard & Kozulin, 2008). This study used qualitative thinking levels to create a design of the learning trajectory where qualitative thinking level consists of 7 indicators that can help students associate between the knowledge they already have and more thoroughly in finding information from the problem. The ability to solve problems is expected to train students in understanding problems, planning strategies to solve problems, solving problems through planning, and checking the results that have been resolved following the level of qualitative thinking levels (Dayat Hidayat et al., 2021). The study aims to develop a learning trajectory on the topic of three-dimensional material geometry to support students' qualitative thinking skills with the RMT approach.

METHODS

Methods In this study, the type of research is qualitative research design research methods type validation studies are used. The focus in this research is to develop a learning trajectory by creating Hypothetical Learning Trajectory (HLT) three-dimensional material to support students' qualitative thinking skills with the RMT approach. The learning trajectory is developed from two problems created where these problems contribute to building micro teaching in third-dimensional learning. HLT is based on three components, namely the purpose of learning, description of activities, and student expectations of the learning process (Gravemeijer & Van Eerde, 2009)

This research is conducted through 3 stages, namely: (1) Preparation Phase, (2) Design experiments consisting of preliminary pilot experiment and teaching experiment, (3) Retrospective analysis (Gravemeijer & Van Eerde, 2009). When analyzing the design that has been made, an assessment of the predictions that have been made with actual learning occurs and is revised according to the results so that the results of the learning trajectory are obtained.

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Procedure of Design Research

First, at the Preparation Phase stage, researchers make problem formulations, determine schools and research subjects review the literature on the research to be done, create learning devices in the form of LKPD and HLT and make problem cards and RPP of third dimensional material in class XII using the RMT approach. HLT designed by researchers in the form of alleged student work on the LKPD given.

The second stage is the Design Experiment stage where this stage is divided into 2 stages, namely pilot experiment and teaching experiment. Pilot experiment is conducted to conduct trials on HLT made to non-subject class students. The trial was conducted to see the sequence of LKPD that was designed and can be developed again to be better for the preparation of the next stage of teaching experiment. Then, this revision stage is conducted regarding HLT that have been designed to adjust to the results of work that has been done by non-subject class students.

In HLT that has been designed, there are activities, learning objectives, qualitative thinking level indicators, and predictions of activities carried out by students. The activity carried out is in the form of 2 problems contained in the LKPD given. Where the purpose in the first problem and the second problem given is that the student can explain the concept of point to line distance through LKPD properly and correctly, students can describe the distance point to line through LKPD properly and correctly, students can describe distance point to line through LKPD properly and correctly, students can determine the distance point to line through LKPD well and correctly. The indicator used is a qualitative thinking level indicator of the RMT approach consisting of 7 indicators, namely, visualizing, labeling, decoding, encoding, searching systematically to gather clear and complete information, use of more than one source of information, and comparing.

Furthermore, on HLT will be designed the predictions of student activities towards the work of LKPD three-dimensional material. In LKPD, there are 2 problems provided where the problem is made according to qualitative thinking level indicators. In the first problem, there is a visualizing indicator where students can describe the building of the cube room precisely, and less precisely because maybe students will describe with sides that are not the same length. Students can also draw the projection line from point X to the KR line i.e. point O, and also there is something that is not right in the placement of the line that is not equally large. The last guess is, students can create pictures, students create pictures but they are incomplete, and students do not create images of RSX, XOR, KPX, and XOK elbow triangles. Furthermore, on the labeling indicator, students can put a name on the klmn cube room building. PQRS is in the right problem, and there is also something that is not right in determining the location of the given name, the last guess is that students can give a name at the midpoint of the PS line segment, namely point X.

The next indicator is decoding, designed to suggest that students can identify that the rib length on a given problem is the same and also that students can identify, but less precisely where, there may be students who forget that the nature of the cube wake has all the same length of sides. In the encoding indicator, there are two

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predictions that students can know and cannot know that the point in the middle of the PS line is point X. The next guess is that students can identify and there are students who cannot identify that from point X to the KR line will be perpendicular to the XO line. The last guess of this indicator is that students can identify exactly that point X to the KR line represents the distance from point to line and there are students who can identify that point X to line KR represents the distance from point to line but is less precise in giving reasons why it is called point to line distance.

Next is the search indicator, searching systematically to gather clear and complete information, there are 6 predictions that students write information about the size of the cube room building and the known point of the problem provided completely, and there are students who write information but incomplete. The student also determines the plan of problem solving i.e. draws a line from point X to R and describes ΔRSX and ΔXOR and the student determines the plan by drawing a line from point X to R but he describes another triangle, not ΔRSX and ΔXOR. Students will solve problems about the distance from point X to the KR line precisely but there are students who solve problems less precisely in steps and results obtained.

On indicators of the use of more than one source of information, students can use the concept of equilateral foot triangles as well as concepts from the Pythagorean theorem, some students cannot use the concept of equilateral foot triangles but can use concepts from the Pythagorean theorem, and there are also students who cannot use the concept of equilateral triangle and concept of the Pythagorean theorem. The last indicator is comparing where students can conclude correctly that the distance of point X to the KR line by using the RX line is equal to the distance of point X to the KR line by using the KX line and some students can conclude, but less precisely, there may be errors in terms of understanding so that the answers obtained are less precise.

Just like the first problem, in the second problem, there are also predictions of students about the work that has been done. On the visualizing indicator, students can describe the building of the cube space precisely, and some students are less precise, students will describe with sides that are not the same length. Students can describe the triangle build obtained from drawing a line from point P to point R and point Q some students can describe the triangle build obtained from drawing a line from point P to point R and point Q but are less precise to draw the line thus does not produce a triangle, and the student cannot describe the triangle build obtained from drawing the line from point P to point R and point Q. The next indicator is labeling; students can put a name on the *back* cube room building. EFGH is in the right problem, some students can give the name to the cube but are not precise in determining the location of the given name. The last guess is that students can give names at the midpoint of the CD line segment, EH line segment, and BF line segment.

The next indicator is decoding, students can identify that the rib length on a given problem is the same and also the student can identify, but not exactly where, there may be students who forget that the nature of the cube wake has all the same rib length. Next on the encoding indicator, It is alleged that students can know and there are

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students who do not know that the point is in the middle of the CD line. BF, and EH are points P, R, and Q. The next guess is that students can identify and there are students who cannot identify that from point P to point R and to point Q there will be a triangle.

Furthermore, the indicator of searching systematically to gather clear and complete information, there is an prediction that students write information about the size of the cube room building and the known point of the problem provided and students write the information incompletely. Students also determine the problem solving plan that describes ΔFEQ, ΔQFR, and ΔPQR And there are students who are not precise in determining the plan where students will forget one triangle to be depicted and directly to another. Furthermore, students can solve the problem of the distance from point P to the QR line and the distance from point S to the TQ line precisely and there are students who are not right in the steps and results obtained. The last guess in this indicator is that students will look back at the triangle and size in the cube and use the concept of a cube where the size of the cube is the same to answer the problem.

On indicators of the use of more than one source of information, students can use the concept of equilateral triangles and also concepts of the Pythagorean theorem, there are students who cannot use the concept of equilateral triangles but can use concepts from the Pythagorean theorem, there are also students who cannot use the concept of equilateral triangle and concept of the Pythagorean theorem, and students can use the concept of a point position. The last indicator is comparing, made 2 predictions that students can identify that the length from point P to the QR line is equal to the length from point S to the TQ line and there are students who are not precise in identifying it, there may be errors in terms of understanding so that the answers obtained are not appropriate.

In the teaching experiment stage, the revised HLT will be tested on the class that is the subject of the study. Teaching experiments are carried out to see the learning trajectories that have been made and comparisons are made to actual learning.

The last stage is the Retrospective analysis stage. At this stage the researcher develops the learning trajectory that has been obtained in the previous stage. At this stage, the HLT is used as a guide and reference to analyze all the data collected during the pilot experiment and teaching experiment stages. The learning trajectory is developed by comparing the created HLT with the actual learning. The conclusions from the analysis carried out at this stage are used to answer the problem formulation. The subjects in this study were class XII students of SMA Negeri 1 Palembang in the academic year 2021/2022. The students who became the focus of the research subject consisted of 6 students with high, medium, and low abilities.

RESULTS AND DISCUSSION

Learning is done face-to-face and online. Face-to-face learning is carried out with students who have even absent numbers and online learning through zoom for students who have odd absent numbers. This research produces a learning trajectory related to three-dimensional material using the RMT approach at the level of qualitative thinking. This research was conducted by students of class XII

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MIPA 7 SMA Negeri 1 Palembang and the researcher will act as a model teacher and the subject teacher will act as an observer assisted by 2 research colleagues. At the time of learning, the LKPD which consists of 2 problems related to three-dimensional material is used based on the learning trajectory that has been made using the level of qualitative thinking.

Problem 1

All figures Students conduct face-to-face and online learning using zoom for 60 minutes, where the zoom link will be sent via a telegram group. During the learning process, the researcher as the model teacher started the lesson by orienting the greetings followed by praying together, then the model teacher did apperception by asking a question about the three-dimensional material that the students had learned. Next, the teacher displays the PPT which discusses the three-dimensional material and the teacher explains the material to all students in the class and zoomed in. Students listen, understand, and record information from the teacher's explanation. The explanation of the three-dimensional material is divided into 2 sub-chapters, namely the point-to-point distance and the point-to-line distance. In the sub-chapter the point-to-point distance is explained by the researcher, and in the sub-chapter the point-to-line distance is explained by a research colleague (Figure 1).

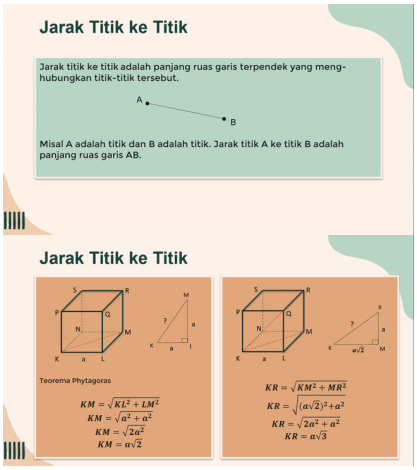

Figure 1. Three dimensional problem about distance

After the teacher finished explaining the material, the teacher distributed LKPD which contained 2 problems and had to be done by students for 25 minutes. The LKPD given is in line with the HLT made which contains the students' assumptions when working on the LKPD (Figure 2).

Given a cube KLMN.PQRS with side length 4 cm. X is the midpoint of PS.

a. Draw the shape of the space in question from the problems given above!
b. Write down any information you get from the problem above!
c. From the problem above, name the sides that have the same distance!
d. If a line is drawn from point X to point R, what is the distance from point X to line KR? Determine the solution!
e. If a line is drawn from point X to point K, is the distance from point X to line KR still the same as before? Explain!
f. Can the distance from point X to line KR be said to be the distance from point to line? Why?

Figure 2. LKPD Problem 1

The LKPD that has been done will be answered by discussing in class, for students who study online it will be collected through the *google form* provided by the teacher. The following are the results of students' answers to the LKPD given.

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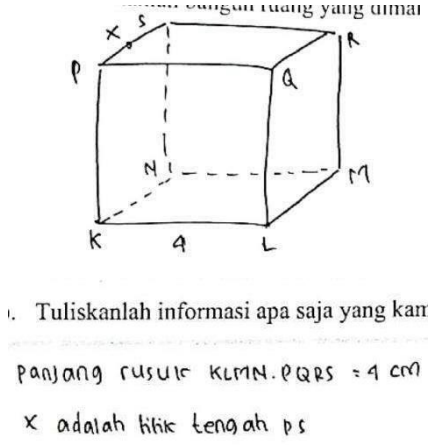

Figure 3. Completion of Problem 1A

Based on the outcome of the work of some students (one of student's work shown is Figure 3), it is seen that the students described with appropriate and gave the name of a point on the build space of the cube but there are students who are less proper to give the name on the dots. The student gave the name to put it on the field of the future, not into the field below. Answers of the image that has been done of students included in visualizing where the students can describe the build space of the cube and labeling students and gave the name of at the point referred to in the problem. There are students who can complete visualizing but cannot complete the labeling.

Furthermore, there are also some student answers related to the problem, namely writing the information on the problem. The answer is included in decoding and searching systematically to gather clear and complete information. On decoding, the students will mention the sides the same from waking up the space and on searching systematically to gather clear and complete information, students will write what information is known of a given problem. Visible differences of the answers written by the students. There is a student writes in full about the sides of the cube, as well as sides PX and SX where X is a point located in the middle of the line PS.

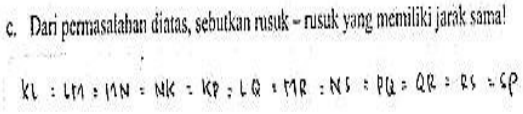

Figure 4. Completion of Problem 1C

Furthermore, students' answers related to the problem are also seen, namely writing the sides that have the same distance on the cube shown in Figure 4. The answers from these students are included in the decoding. In the picture above, it can be seen that students understand that a cube is formed from a square where the square has all the same sides so that the student writes down all the sides on the cube that are equidistant.

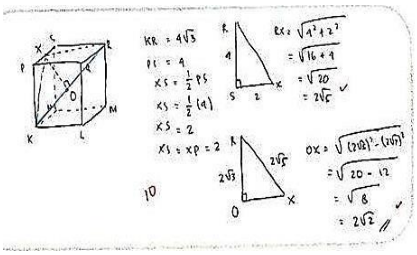

Figure 5. Completion of problem 1D

Figure 5 is one of student's answer for problem 1D, based on problem d, visualizing indicators are again visible when students describe building triangles to solve problems. There are also some answers to students related to the problem, namely looking for information from the problem. At that time, students enter the process of searching systematically to gather clear and complete information where students will look for information that will be used during the process of solving problems. The information obtained by students is different. There are students who obtain the information appropriately and there are students who are unable to obtain such information.

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Next, students will plan a way to solve the problem. There are several different answers from students, namely describing the triangle that will be removed from the cube room building, making an elbow triangle to look for the value asked and there is also a triangle that makes any triangle to look for the value. And the last stage to solve it is to use the Pythagorean theorem and substitute the value appropriately to solve it so that the results obtained are precise and there are also those who use the Pythagorean theorem but not precisely in substituting values into the Pythagorean theorem so that the results obtained are not accurate.

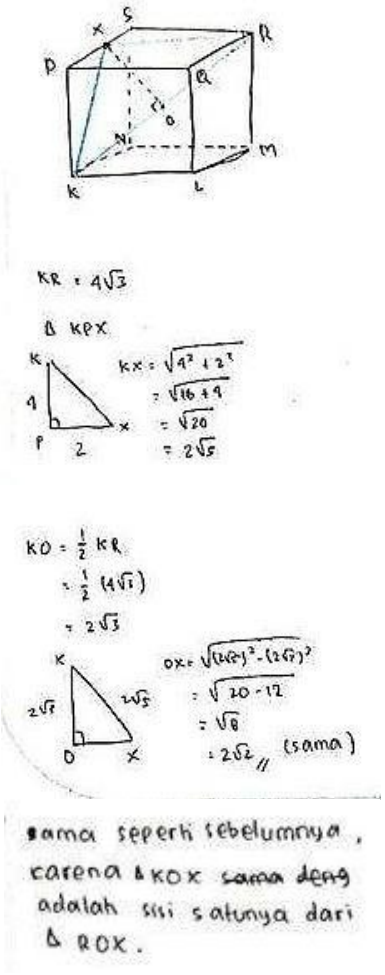

Figure 6. Completion of problem 1E

In Figure 6, there are several different student answers in solving it. This is included in using more than one source of information and comparing. Some use the concept of the equilateral triangle of the triangle obtained by drawing a line on the cube and there are also students who seek the solution by looking again for distance from the question using the Pythagorean theorem.

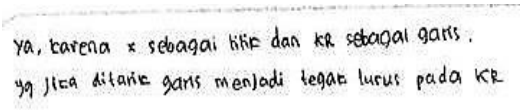

Figure 7. Completion of problem 1F

The last question in problem 1 (Figure 7) given to students is to identify the reason why the problem can be said to be the distance of the point to the line. This problem is intended so that students can understand what is a concept from point to line, not just solve the given problem. Seen in the image above the student gives an answer according to the concept of striped point distance. This process can also be referred to as encoding where students write about what they understand about the distance of the point to the line.

Problem 2

LKPD problem 2 is done together with problem 1. After problem 1 is discussed, students will collect LKPD containing answers to problem 2 in Figure 8.

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Given a cube ABCD.EFGH which has an side of 8 cm. Point P is the midpoint of line CD. Point Q is the midpoint of the line EH. Point R is the midpoint of line BF.

a. Draw the shape of the space in question from the problems given above!
b. Write down what information you get from the problem above. If not, why?
c. From the problem above, name the sides that have the same distance!
d. How to determine the distance from point P to the QR line? Determine the solution!
e. If point S is the midpoint of AB and point T is the midpoint of CG. Is the distance from point P to the QR line the same as the distance from point S to the TQ line? If the same, give reasons!

Figure 8. LKPD problem 2

Figures 9-12 are the results of students' answers to problem 2. Based on the results of their answer (Figure 9), it is seen that the students described with a proper cube of the mean of a given problem. Along with describing the cube, the students also gave the name of a point on the build space of the cube so that students meet the process of visualizing and labeling.

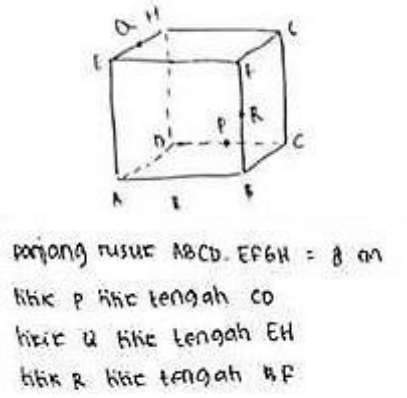


Figure 9. Completion of problem 2A and 2B

Furthermore, it is also seen that students write down the information in the problem. The answer is included in searching systematically to gather clear and complete information. In the indicator searching systematically to gather clear and complete information, students will write down what information is known from the problem provided. Seen in the answers that have been done, students write down known information such as sides, and known points of problems given.

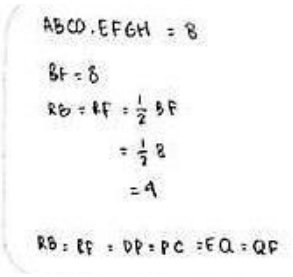
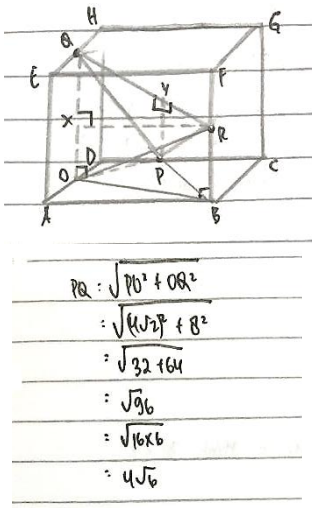


Figure 10. Completion of problem 2C

Furthermore, the students' answers (Figure 10) about the distance between the sides are the same on the cube. This is included in the decoding. The student mentions the sides that have the same distance on the cube and at the same time the student also understands that the cube is formed from a square where the square has all the same sides.



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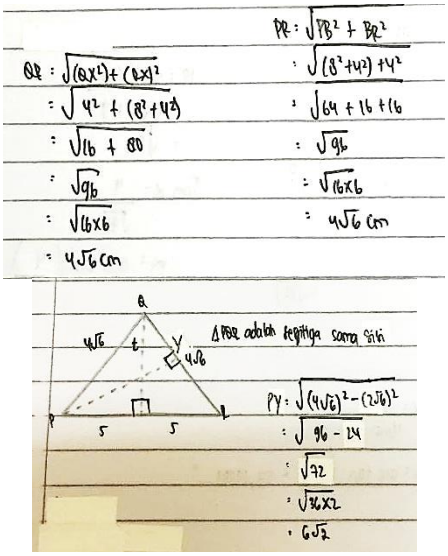

Figure 11. Completion of problem 2D

Based on Figure 11, it can be seen that the students' answers are related to seeking information from these problems. At that time, students enter into the process of searching systematically to gather clear and complete information in which students will look for information that will be used during the process of solving problems. Students will also plan how to solve the problem by drawing a triangle obtained from the shape of a cube. At the time of completing it, students must know that the acquired triangle is an equilateral triangle so that students can use the Pythagorean theorem and substitute the values appropriately to get the right results. In the image above, it is seen that students complete the indicators of searching systematically to gather clear and complete information.

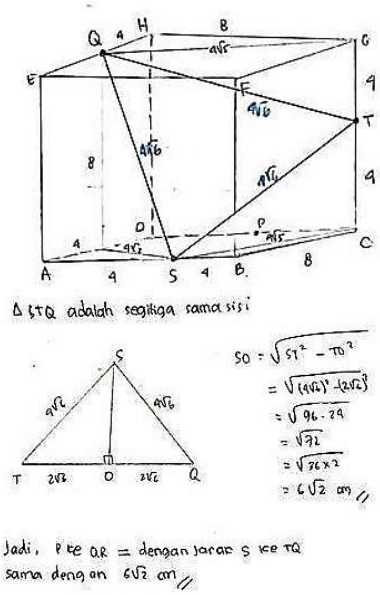

Figure 12. Completion of problem 2E

In Figure 12, students' answers when completing them included using more than one source of information and comparing. Students answer the problem by using the concept of an equilateral triangle from the triangle obtained, students also do a re-calculation to confirm the answer using the Pythagorean theorem. But there are also students who immediately knew that a triangle obtained an equilateral triangle without describing the back of the triangle.

Test Questions

Written tests are given to students after all the learning processes are carried out. This test question consists of 2 questions where 1 question is about the distance of the point to the line and 1 question is for the distance of the point to the plane. In this study, the researcher only used question 1. The test questions were given in pdf form to students via the telegram group and students would upload them via the google form that had been provided with the specified time limit. Test

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questions are given to analyze student answers following the indicators of the level of qualitative thinking. After carrying out the learning activities, the data that has been obtained from each activity are in the form of LKPD, HLT, test results, and interviews regarding the written tests that have been carried out. The researcher analyzed the data obtained with the learning trajectory that had been designed in the form of HLT which contained students' assumptions about the work on the LKPD given. The formulation of the problem that has been made by researchers regarding the development of the learning trajectory is answered where in the learning trajectory that has been obtained, the desired level of students' qualitative thinking appears in student work. The HLT that has been designed, then revised at the pilot experiment stage and tested again at the teaching experiment stage, gets results in the form of a learning trajectory where the HLT is compared with actual learning (Actual Learning).

CONCLUSION AND SUGGESTION

Based on what has been described above, it can be concluded that the learning trajectory produced in research related to three-dimensional material using the RMT approach with qualitative level of thinking can make students solve problems in detail. Students carry out the stages of describing the desired shape, giving names, and thinking about plans to be carried out to solve the problems given appropriately. The students also explain their understanding of the concepts of the material being taught so that students grasp the material. The following are the stages of students' understanding of the problems given regarding the three-dimension. It can be ensured that students understand the material and achieve the learning objectives. The learning trajectory also meets qualitative thinking level indicators, especially visualizing and labeling indicators.

Recommendation for the next research related to Rigorous Mathematical Thinking (RMT) is to emphasize in the evaluation of student's learning and how to assess indicators in the achievement of learning using this approach.

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