AKSIOMA: Jurnal Program Studi Pendidikan Matematika
Volume 11, No. 3, 2022, 2333-2344
ISSN 2089-8703 (Print)
ISSN 2442-5419 (Online)

DOI: https://doi.org/10.24127/ajpm.v11i3.5061

DEFRAGMENTING STUDENTS' THINKING STRUCTURE IN SOLVING MATHEMATICAL ARGUMENTS

Nonik Indrawatiningsih[1*] , Andika Setyo Budi Lestari[2]

[1*] Universitas Negeri Surabaya, Surabaya, Indonesia
[2] Universitas PGRI Wiranegara, Pasuruan, Indonesia
[*]Corresponding author
E-mail: *nonikindrawatiningsih@unesa.ac.id[1*)]*
andikalestari123@gmail.com [2)]

Received 19 March 2022; Received in revised form 10 August 2022; Accepted 29 September 2022

Abstrak

Struktur berpikir adalah representasi dari proses berpikir yang berupa alur penyelesaian masalah yang dilakukan oleh seseorang ketika ia menyelesaikan suatu permasalahan. Banyak siswa yang melakukan kesalahan dalam menyelesaikan masalah argumen matematika. Siswa sering melakukan kesalahan dalam menyusun premis menggunakan simbol matematika, membuat model matematika, dan membuat tabel kebenaran untuk membuktikan argumen yang valid. Selain itu, mereka juga kurang memahami sistem modus ponens, modus tollens, dan silogisme. Salah satu cara yang dapat dilakukan untuk mengatasi kesalahan tersebut adalah dengan melakukan defragmenting struktur berpikir. Penelitian ini menggunakan metode kualitatif dan bertujuan untuk mendeskripsikan tentang kesalahan struktur berpikir siswa dalam menyelesaikan masalah argumen matematika serta upaya defragmentingnya. Penelitian ini dilakukan pada siswa Kelas X Program IPA pada salah satu SMA Negeri di Pasuruan. Subjek penelitian dipilih berdasarkan tiga kriteria, yaitu tingkat kesalahan rendah, sedang, dan tinggi. Kesalahan struktur berpikir siswa ditelusuri dari hasil think out loud siswa selama proses penyelesaian masalah argumen matematika. Data yang diperoleh kemudian dikodekan dan dijadikan dasar untuk menggambarkan proses defragmenting yang dilakukan. Berdasarkan hasil penelitian dapat disimpulkan bahwa kesalahan prosedural siswa dalam menyelesaikan masalah argumen matematika, yaitu menentukan nilai dari suatu persamaan, membuat bentuk model argumen, dan membuktikan argumen valid. kemudian peneliti melakukan Defragmenting dengan cara memberikan scaffolding untuk memperbaiki struktur berpikir siswa dalam menyelesaikan masalah argumen matematika.

Kata kunci: Defragmenting; struktur berpikir; argumen matematika; scaffolding.

Abstract

The structure of thinking is a representation of the thought process in the form of a problem-solving flow that is carried out by a person when he resolves a problem. many students make mistakes in solving mathematical argument problems. Students often make mistakes in constructing premises using mathematical symbols, making mathematical models, and making truth tables to prove valid arguments. In addition, they also lack understanding of the modus ponens, modus tollens, and syllogism systems. One way that can be done to overcome these errors is to defragment the structure of thought. This study uses qualitative methods and to describe the students' erroneous thinking structure in solving mathematical argument and the defragmenting efforts. The students of 10th Grade of high school in Pasuruan, East Java, Indonesia, were involved as research subjects. They were selected based on three criteria, namely low, moderate and high level of procedural error. The activity of 'think out loud' was used to observe the errors made by students in solving mathematical argument. The data obtained from this activity were codified and later used as a basis to perform the defragmenting process. Based on the findings of this study, it can be concluded that procedural errors in solving mathematical argument are in the form of error in determining the value of x from an equation, modeling an argument, and proving a valid argument. Defragmenting was done using scaffolding approach to improve students' thinking structure in solving mathematical problems.

***Keywords**: Defragmenting; thinking structure; mathematical argument; scaffolding.*

DOI: https://doi.org/10.24127/ajpm.v11i3.5061

INTRODUCTION

Problem solving is a very important component of mathematics curricula because in this process, students might have experience using their acquired knowledge and skills in solving non-routine problems (Das & Chandra, 2013; Bahrudin, Indrawatiningsih, & Nazihah, 2019). As they obtain such new experience and knowledge, it might lead to changes in behavior. Learning mathematics involves reasoning in patterns and traits, mathematical manipulation in making generalizations, compiling evidence, or explaining ideas and statements in mathematics (NCTM, 2000). In term of logic in mathematics, reasoning activities are extensively carried out to state various statements. Reasoning involves abstraction that is included since this activity entails conclusion drawing from one proposition or more. Henceforth, reasoning activity is called argument. Each argument consists of particular statements and further statements that logically accompany those statements (Conner et al., 2014b; Indrawatiningsih, 2018). According to Van Ness & Maher (2018) argument is emphasized on making logical relations, reasoning or conclusions between propositions.

In solving problem in the form of argument, students frequently face difficulties in proving an argument. It is mainly sourced from the complexity in turning written words into mathematical symbols (Salma & Sherwin, 2012). Furthermore, such difficulties may lead to errors in proving valid arguments. The mistakes can be made in writing symbols, drawing conclusions, processing and having misconception (Pape, 2012). Based on (Indrawatiningsih, 2018) three types of error in solving mathematical problems are conceptual error, procedural error, and technical error the mistakes often made by most students are categorized in procedural error.

The results of the preliminary test divulge the fact that most students have not been able to prove valid arguments. It is indicated by their worksheets in which there were many procedural errors in proving valid arguments. These include mistakes in making premises using mathematical symbols, making mathematical model, and making truth tables to prove valid arguments. In addition, they have inadequate understanding about the system of modus ponens, modus tollens, and syllogism in drawing conclusions.

In learning mathematics, the act of thinking is involved since it requires mental activities in doing so. This process is an activity of learning. In fact, learning activities are not merely about listening to the material, writing the material, and doing the assignments, but also mental processes that occur in the mind. Thinking process starts from receiving, processing, and storing data in memory to recalling it when needed. Students' thinking processes are determined by the capacity of the thinking structure to a given problem (Wulandari & Gusteti, 2021). The thinking structure is a representation of the thinking process, namely a flowchart of problem solving carried out by a person in resolving a problem.

In general, procedural error illustrates students' inability in correlating their knowledge with a problem related to mathematical argument (Bulent et al., 2016). It is the result of the non-conformity between the students' thinking structure and the given problem (Supiarmo, 2021). One of the solutions to overcome this issue is by defragmenting the thinking structure. In

AKSIOMA: Jurnal Program Studi Pendidikan Matematika ISSN 2089-8703 (Print)
Volume 11, No. 3, 2022, 2333-2344 ISSN 2442-5419 (Online)

DOI: https://doi.org/10.24127/ajpm.v11i3.5061

the present study, it is intended to help research subjects in re-structuring their thinking structure.

Defragmenting the thinking structure is a rearrangement of students' thinking structure in relation with the errors made in solving problems, which can be done through the scaffolding process (Sakif, 2014; Wibawa, Nusantara, Subanji, & Parta, 2018). Scaffolding is defined as the provision of assistance/support for a learner during the initial stages of learning until the learner is able to solve the problems. The present study aims to provide description about defragmenting the students' thinking structure in solving mathematical argument problems based on the procedural errors made by students. The present study aim to describe students' erroneous thinking structure in solving mathematical argument and the defragmenting efforts. Two research questions (RQs) have been submitted to meet the research objectives of this study i.e the first research question (RQ1): where the students' mistakes in solving mathematical arguments?. The second research question (RQ2): how to defragmenting students' thinking structures in solving mathematical arguments?.

METHOD

Data Collection

The present study is qualitative descriptive study. It aims to systematically analyze and provide facts of a problem (Creswell, 2012). It investigates the errors made by students and formulate how defragmenting students' thinking structure in relation with mathematical arguments, particularly in arranging valid argument. Primary data were obtained through test and in-depth interview method. Written test contains the problems of mathematical argument. Subsequently, in-depth interview is intended to validate the data.

Participation

As many as 36 students of class X nature sciences program of a state senior high school in Pasuruan, East Java, Indonesia, were involved as research subjects. They had studied the material of mathematical arguments and were assumed to recall the material during the research. The selection also considered the procedural errors made by students when solving problems of mathematical arguments and students' communication skills in order to ease the disclosure of thinking processes. Students were required to solve problems individually by writing out the steps clearly.

Several criteria were set for the selection of research subjects, including: 1) students made one type of procedural error (low level), 2) students made two types of procedural error (moderate level), and 3) students made more than two type of procedural error (high level). Subsequently, the selected research subjects were interviewed.

Data Analysis

In the resent study, interviews aim to explore and clarify the problems raised by the research subjects. Therefore, an unstructured interview approach was preferred. Students had to 'think out loud' by conveying their thinking structure during the problem solving activity. Subsequently, the process of defragmenting the thinking structure of the subjects in association with the procedural errors was carried out using scaffolding approach. Furthermore, data analysis was performed using Miles and Hubberman's flowchart, namely (1) data reduction, (2) data display, and (3) conclusion drawing. The problems of mathematical

ISSN 2089-8703 (Print)
ISSN 2442-5419 (Online)

DOI: https://doi.org/10.24127/ajpm.v11i3.5061

arguments given to students are presented in bahasa. The following question is the translation of the question in English.

"Last month, the average temperature was 40°F. *Meanwhile, the actual temperature could be higher or lower by 10*°F.

a. Model this situation in an absolute value equation.

b. Use this equation to determine the hottest and the coldest temperature.

c. *Make a valid argument based on the above answers."*

RESULT AND DISCUSSION

Based on the results of tests that involved 36 students, it can be stated that the majority of students make procedural errors in solving mathematical arguments. Specific number of students in accordance with the level of procedural errors and communication skills is presented in Table 1.

Table 1. Number of students based on level of procedural error and communication skills

		Level of procedural error		
		Low	Moderate	High
Communication Skills	Good	2	2	7
	Less	2	5	18

Furthermore, three students are determined as research subjects with the criteria: Subject 1 (S1) is student with low level error; Subject 2 (S2) is student with moderate level error, and Subject 3 (S3) is student with high level error. The thinking structure of these subjects in dealing with exponential inequalities as well as the defragmenting process is elaborated below.

The Thinking structure of S1 in Solving Mathematical argument

The differences or changes in student's thinking structure in solving mathematical argument in the pre- and post-defragmenting process are detailed in Figure 1.

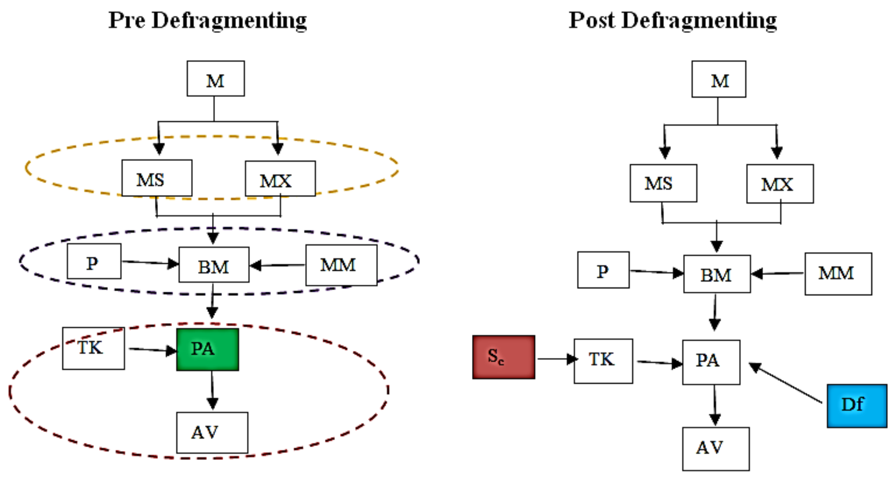

Figure 1: The Thinking structure of S1 in Pre- and Post-defragmenting

ISSN 2089-8703 (Print)
ISSN 2442-5419 (Online)

DOI: https://doi.org/10.24127/ajpm.v11i3.5061

The structure of student's thinking structure in solving mathematical argument is illustrated in codified boxes with an explanation of each code. This explanation is presented in Table 2.

Table 2. The codes in the thinking structure

Code	**Description**
M	Problem
MS	Model of situation
MX	Value of equation
P	Making premises with mathematical symbol
BM	Modeling an argument
MM	Forming an argument
PA	Proving a valid argument
TK	Truth Table
AV	Valid argument
Df	Defragmenting
S_C	Scaffolding
	First step
	Second step
	Third step

Figure 1 shows the procedural error made by S1. This error is marked by the green code, namely proving an argument (PA), as performed in the third step of proving a valid argument. S1 failed in proving the argument previously made. This argument could not be validated using the truth table. The truth table contained several incorrect points, leading to an error in interpreting the argument. Subsequently, the defragmenting process was carried out to re-structure student's thinking structure.

During the interview, S1 was inquired about the truth table that revealed the mistake in writing the mathematical symbols as illustrated in Figure 2. After realizing the mistake, S1 began to get puzzled since the first premise is false, while the second and the conclusion are true. Can such argument be valid? Scaffolding was then given by questioning S1 about the definition of valid arguments. After a few minutes, S1 finally figured out that all valid arguments must be true so that the conclusion is true. The following is the conclusion of the interview between the researcher and S1. Students still have difficulty in determining valid or invalid arguments. Then the research provides scaffolding in the form of questions that lead to the definition of the argument. After students reread the definition of argument while thinking, the student understands that an argument is said to be valid if all the premises are true and the conclusion is also true.

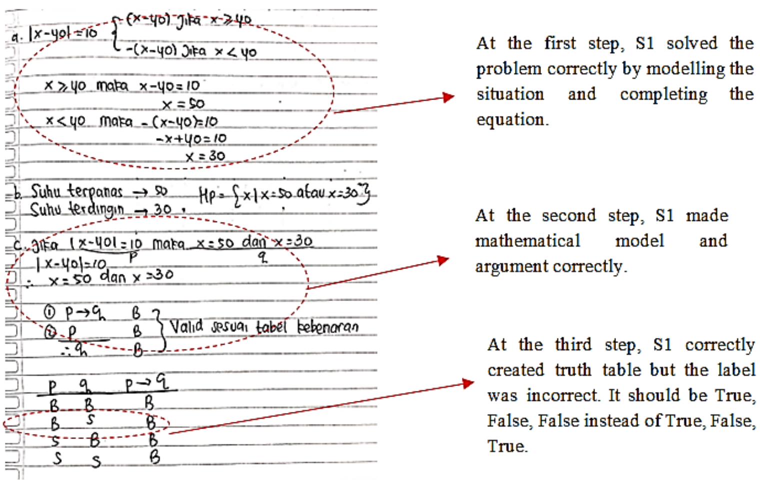

Figure 2. S1's Worksheet

ISSN 2089-8703 (Print)
ISSN 2442-5419 (Online)

DOI: https://doi.org/10.24127/ajpm.v11i3.5061

The Thinking structure of S2 in Solving Mathematical argument

The differences or changes in student's thinking structure in solving mathematical argument in the pre- and post-defragmenting process are detailed in Figure 3. Figure 3 demonstrates S2 made two types of procedural error as marked by green cod, namely PA and BM. The mistakes are identified in the second and third steps, i.e., modeling an argument and proving a valid argument. S2 failed in both modeling the argument appropriately and proving it as a valid one. Despite S2 had successfully made premises with mathematical symbol, the equation was incorrect as illustrated in Figure. 3. Subsequently, defragmenting process was performed to re-structure the student's thinking structure through scaffolding. It was done by re-examining the forms of argument. After recalling the forms, S2 began to understand how to form arguments. Nevertheless, S2 was confused how to select appropriate form. Second scaffolding was done by giving directions about various forms of argument, namely modus ponens, modus tollens, and syllogism. It aimed to strengthen student's understanding for finding a solution.

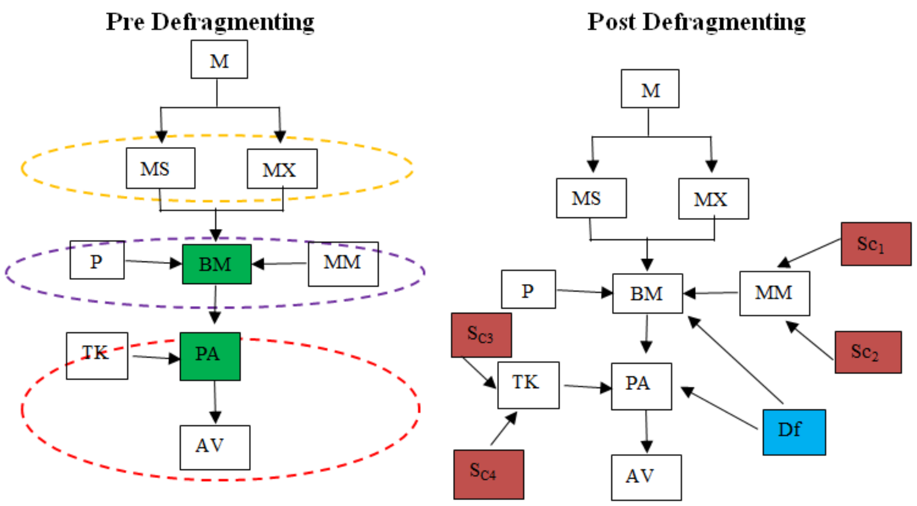

Figure 3. The Thinking structure of S2 in the Pre- and Post-defragmenting

Another mistake was made in the third step at which S2 failed in proving a valid argument. Third scaffolding was done by investigating the approaches used to prove a valid argument. In addition, S2 made a mistake in creating the truth table as it only consisted of one row and each premise and conclusion is all true (Fig. 4). Subsequently, fourth scaffolding was carried out to improve student's thinking structure by questioning the precise number of rows in the truth table. S2 revised the error by making a correct truth table and filling it with the premises. This revision led the student to find solutions related to valid arguments. The following is the conclusion of the researcher's interview with S2 The researcher asked to explain the steps in making an argument model. After that, students make an argument model and explain again what has been written. students make mistakes in writing the word "and" in a premise. But the truth is that it should use the word "or" in the premise.

AKSIOMA: Jurnal Program Studi Pendidikan Matematika ISSN 2089-8703 (Print)
Volume 11, No. 3, 2022, 2333-2344 ISSN 2442-5419 (Online)

DOI: https://doi.org/10.24127/ajpm.v11i3.5061

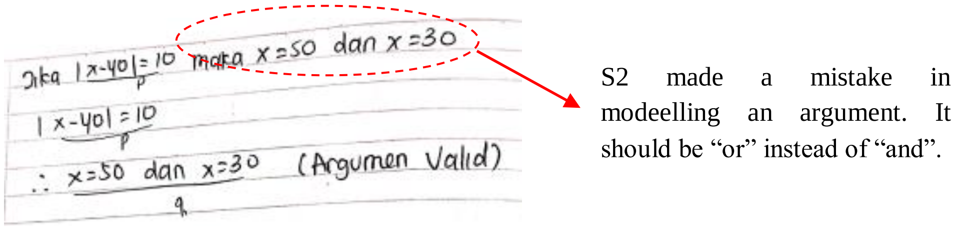

Figure 4. S2's Argument Model

The following are the conclusions of the researcher's interview with S3: students can explain the truth table of the arguments presented. However, the table made is only 1 row (in Figure 5) and has concluded that the premise and conclusion are true. Then the researcher gave scaffolding in making the truth table and asked to make a tree diagram in determining the number of rows in the truth table. From this, the students realized that the truth table made was wrong so that the truth table was corrected.

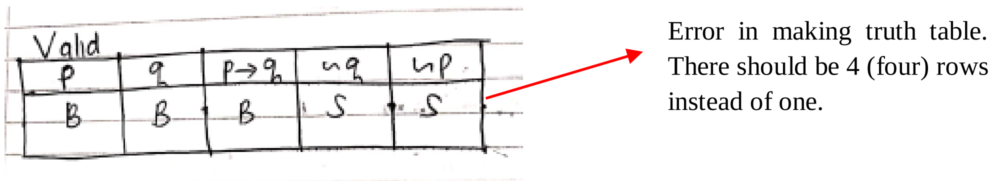

Figure 5. S2's Truth Table

The Thinking structure of S3 in Solving Mathematical argument

The differences or changes in the student's thinking structure in solving mathematical argument in the pre- and post-defragmenting process are presented in Figure 6. Figure 6 shows three types of procedural error made by S3 in solving the problem.

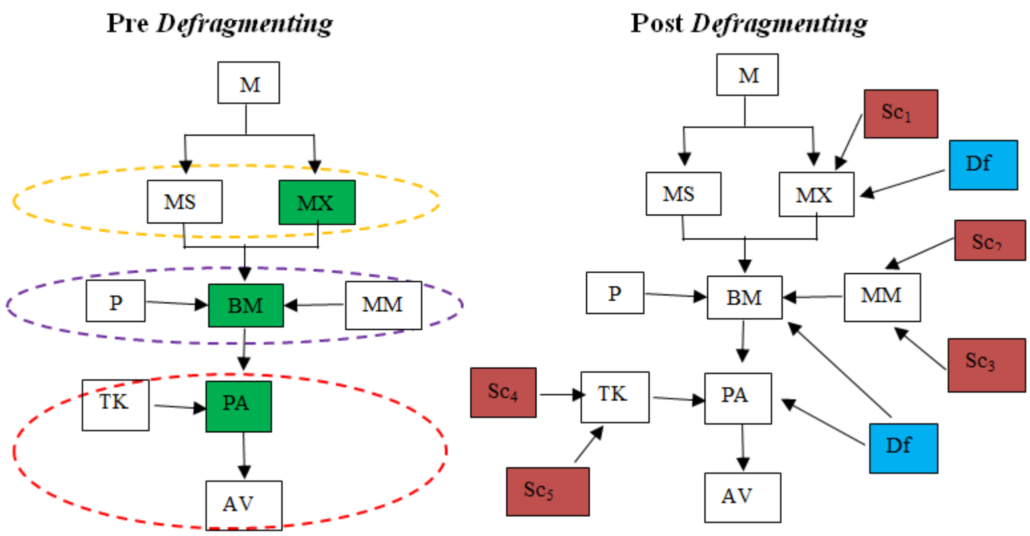

Figure 6. The Thinking structure of S3 in Pre- and Post-defragmenting

AKSIOMA: Jurnal Program Studi Pendidikan Matematika
Volume 11, No. 3, 2022, 2333-2344

ISSN 2089-8703 (Print)
ISSN 2442-5419 (Online)

DOI: https://doi.org/10.24127/ajpm.v11i3.5061

Based on Figure 6, The errors are indicated by the green code, namely PA, BM, and MX. They are partly identified in the first, second, and third steps, i.e., determining the value of an equation, modeling an argument, and proving valid argument. The error is made by S3 in determining the value of x from equation $|x - 40| = 10$, i.e., $x = 50$ or $x = -50$ (Fig. 7). The value of x from equation $|x - 40| = 10$ should be $x = 50$ or $x = 30$. In addition, S3 made a mistake in modeling the argument despite the premises were correct (Fig. 8). Consequently, S3 failed in proving a valid argument. The worksheet shows S3 claims the argument is invalid. Therefore, defragmenting aims to re-structure the student's thinking structure. Ultimately, S3 was posed to verify the worksheet for ensuring the final statement.

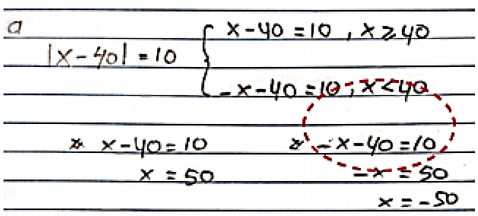

Figure 7. Error in determining the value of x

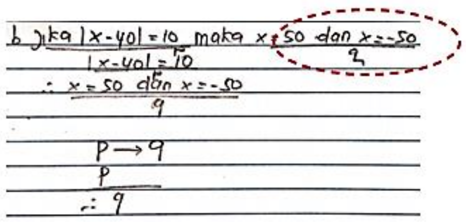

Figure 8. Error the modeling of the argument

First scaffolding was done by investigating how S3 determined the value of x from the equation. The answer was in line with the worksheet. Second scaffolding was done by identifying the absolute value equation at which S3 realized previous incorrect steps in determining the value of x. Third scaffolding was done by recalling how to model mathematical argument. After a few minutes, S3 recalled that the principles of modus ponens, modus tollens, and syllogism could be applied for this purpose. Furthermore, to prove a valid argument, it turned out the truth table was inappropriate as indicated by incorrect premises hence the argument was invalid. Therefore, fourth scaffolding was carried out by scrutinizing the approach to prove a valid argument. In addition, S3 made a mistake in determining the truth of the premises and conclusions. Consequently, the argument is valid (Figure 9). Fifth scaffolding was performed to re-structure the student's thinking structure by posing questions about the truth table in which S3 realized the erroneous symbols in the table. It led S3 to find a solution for proving a valid argument.

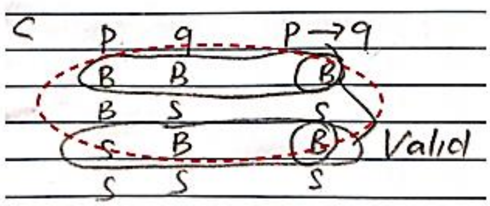

Figure 9. Error in proving a valid argument

Errors caused by incorrectly modeling arguments such as those made by S2 and S3 can be classified into procedural errors. Procedural errors are the most common types of mistakes that are frequently made by students (Karal & Riccomini, 2016). Procedural knowledge plays a second role and becomes a support for conceptual knowledge (Kirshner, 2014). In addition, the most frequent mistakes

DOI: https://doi.org/10.24127/ajpm.v11i3.5061

made by students in logical reasoning and proof lie in proving valid arguments (Indrawatiningsih, 2018). By using truth tables in proving valid arguments, logical reasoning can be improved (Stylianides & Bieda, 2016; Zazkis & Chernoff, 2015). The majority of students are accustomed to using inductive reasoning but not familiar with valid types of proof. In addition, students are used to neglect the basic principles of mathematical logic in developing theory thus they unsuccessfully prove the theorem correctly (Indrawatiningsih, 2018).

In the present study, defragmenting refers to the experts, including (Wibawa et al., 2018). The finding of this study reveals that most students experience procedural errors in solving the problems of mathematical argument. They include errors in determining the value of x in an equation, modeling an argument, and proving valid arguments. Students with low level of error are inclined to make mistakes in determining the x value of an equation. They incorrectly make an argument model and it has implications for the failure in proving a valid argument. It is called repair theory error, where students do not actually understand the given problem. Therefore, students tend to make procedural error (Wibawa et al., 2018). In this context, defragmenting is attempted to improve students' thinking structure through scaffolding approach. It is conducted by questioning the students about the information acquired from the problem. According to (Das & Chandra, 2013), this type of scaffolding require activity prior knowledge by first focusing on what students know and understand. Furthermore, students with moderate and high level of error tend to make mistakes in applying the principles in modeling mathematical argument, i.e., modus ponens, modus tollens, and syllogism.

Defragmenting for students with moderate and high level of error requires longer and more complex process compared to those with low level of error. In addition to higher number of procedural errors, students with moderate and high level of error have a low conceptual understanding needed to solve mathematical argument (Kirshner, 2014); Sakif, 2014).

CONCLUSION AND SUGGESTION

Based on the results of research and discussion in this study, it can be concluded that the present study reveals that in relation with the thinking structure, students mostly make procedural errors. Three types of procedural error are identified, namely: low, moderate, and high level. The low level students make one type of procedural error. The moderate level students make two types of procedural error. The high level students make more than two types of errors.

Low level is identified by the inconsistency in proving valid arguments using truth table. Moderate level is identified by incorrect argument model and inconsistency in proving mathematical arguments using truth table. Meanwhile, high level of error is identified when students are incorrect in determining the value of an equation, modeling an argument, and proving a valid argument using truth table.

Therefore, defragmentation is carried out to re-structure students' thinking structure for solving mathematical arguments appropriately. Scaffolding becomes an approach in defragmenting process. It aims to assist the students for successfully determining the value of an equation,

DOI: https://doi.org/10.24127/ajpm.v11i3.5061

modeling an argument, and proving valid arguments using a truth table. So, the nest result of this study is defragmenting students' thinking structures in solving mathematical arguments can be done by providing scaffolding in the form of question assistance that can lead students to be able to find solutions to their problems.

The findings of this study can affect the students' thinking structure in solving mathematical problems so was done using scaffolding approach to improve students' thinking structure in solving mathematical problems. In this study, the characteristics of research subject have not been taken into account, particularly in relation with the defragmenting process. Therefore, it is suggested for further similar study to investigate the characteristics of research and their relationship with the findings. In this context, the characteristics can be learning style, cognitive style and personality.

REFERENCES

A. Karal, M., & Riccomini, P. J. (2016). General Education Teachers ' Perceptions About Students With. *International Journal of Special Education*, *31*, 23–31.

Bahrudin, M. A., Indrawatiningsih, N. &, & Nazihah, Z. (2019). Defragmenting Struktur Berpikir Siswa SMP dalam Menyelesaikan Masalah Bangun Datar. *Indonesia Mathematics Education (IndoMath)*, *2*(2), 127–140.

Bulent, D., Erdal, B., Ceyda, A., Betul, T., Nurgul, C., & Cevahir, D. (2016). An analysis of teachers questioning strategies. *Educational Research and Reviews*, *11*(22), 2065–2078. https://doi.org/10.5897/err2016.3014

Conner, A., Singletary, L. M., Smith, R. C., Wagner, P. A., & Francisco, R. T. (2014). Teacher support for collective argumentation : A framework for examining how teachers support students ' engagement in mathematical activities. *Educational Studies in Mathematics, 86(3), 401 - 429.* https://doi.org/10.1007/s10649-014-9532-8

Creswell, J. (2012). *Educational Research*. Pearson Education, Inc., 501 Boylston Street, Boston, MA 02116.

Das, R., & Chandra, D. G. (2013). Math Anxiety : The Poor Problem Solving Factor in. *International Journal of Scientific and Research Publications*, *3*(4), 1–5.

Indrawatiningsih, N. (2018). Arguments in Critical Thinking Ability. *International Conference on Science , Mathematics, and Education*, Vol.218. https://doi.org/10.2991/icomse-17.2018.4

Kirshner, D. (2014). JRME Online : Issues : November 2000 : Pages 524-540 Interference of Instrumental Instruction in Subsequent Relational Learning. *JRME, November 2000*. https://doi.org/10.2307/749885

Lee, Kosze & Smith III, J. P. (2009). Cognitive and Linguistic Challenges in Understanding Proving. *Proceedings of the ICMI Study 19 Conference: Proof and Proving in Mathematics Education*, *2*, 2–21.

Legutko, M. (2008). *of Mathematics Teaching Research :* (Issue 226685).

NCTM. (2000). Introduction: Perspectives on Principles and Standards for School

DOI: https://doi.org/10.24127/ajpm.v11i3.5061

Mathematics. *School Science and Mathematics*, *101*(6), 277–279. https://doi.org/10.1111/j.1949-8594.2001.tb17957.x

Pape, S. J. (2012). Middle School Children ' s Behavior : A Cognitive Analysis from a Reading Comprehension Perspective. *Journal for Research in Mathematics Education*, *35*(3).

Sakif, S. (2014). Defragmenting Of Thinking Process Through Cognitive Mapping To Fix S tudent ' s Error In Solving The Problem Of Algebra. *This Paper Has Been Presented at International Seminar on Innovation in Mathematics and Mathematics Education 1st ISIM-MED 2014 "Innovation and Technology for Mathematics and Mathematics Education" Department of Mathematics Education,Yogyakarta State Univ*.

Salma, J., & Sherwin, R. (2012). Students' Difficulties in Comprehending Mathematical Word Problems in English Language Learning Contexts. *International Reseacher*, Vol 1, No.3, 152 - 160.

Stylianides, A. J., & Bieda, K. N. (2016). Proof and Argumentation in Mathematics Education Research. *Journal for Research in Mathematics Education*, Vol. 35, No. 3, pp: 289 - 321.

Supiarmo, M. G. (2021). Defragmenting Student's Thinking Structures in Solving Mathematical Problems on Pisa Model. *(JIML) Journal of Innovative Mathematics Learning*, *4*(4), 167–177.

Van Ness, C. K., & Maher, C. A. (2018). Analysis of the argumentation of nine-year-olds engaged in discourse about comparing fraction models. *Journal of Mathematical Behavior*, *January*, 0–1. https://doi.org/10.1016/j.jmathb.2018.04.004

Wibawa, K A, Nusantara, T., Subanji, & Parta, I. N. (2018). Defragmentation of Student ' s Thinking Structures in Solving Mathematical Problems based on CRA Framework Defragmentation of Student ' s Thinking Structures in Solving Mathematical Problems based on CRA Framework. *IOP Conf. Series: Journal of Physics: Conf. Series 1028*.

Wibawa, Kadek Adi, Nusantara, T., Subanji, & Nengah Parta, I. (2018). Defragmentation of Student's Thinking Structures in Solving Mathematical Problems based on CRA Framework. *Journal of Physics: Conference Series*, *1028*(1). https://doi.org/10.1088/1742-6596/1028/1/012150

Wibawa, Kadek Adi, Payadnya, I. P. A. A., Atmaja, I. M. D., & Simons, M. D. (2020). Defragmenting structures of students' translational thinking in solving mathematical modeling problems based on CRA framework. *Beta: Jurnal Tadris Matematika*, *13*(2), 130–151. https://doi.org/10.20414/betajtm.v13i2.327

Wulandari, S., & Gusteti, M. U. (2021). Defragmentation of Preservice Teacher's Thinking Structures in Solving Higher Order Mathematics Problem. *Journal of Physics: Conference Series*, *1940*(1). https://doi.org/10.1088/1742-6596/1940/1/012099

ISSN 2089-8703 (Print)
ISSN 2442-5419 (Online)

DOI: https://doi.org/10.24127/ajpm.v11i3.5061

Yackel, E. (2002). What we can learn from analyzing the teacher's role in collective argumentation. *Journal of Mathematical Behavior*, *21*(4), 423–440. https://doi.org/10.1016/S0732-3123(02)00143-8

Yackel, E., & Cobb, P. (1996). Sociomathematical Norms, Argumentation, and Autonomy in Mathematics. *Journal for Research in Mathematics Education*, *27*(4), 458. https://doi.org/10.2307/749877

Yackel, E., Rasmussen, C., & King, K. (2000). Social and sociomathematical norms in an advanced undergraduate mathematics course. *The Journal of Mathematical Behavior*, *19*(3), 275–287. https://doi.org/10.1016/S0732-3123(00)00051-1

Zazkis, R., & Chernoff, E. J. (2015). *What makes a counterexample exemplary ? What makes a counterexample exemplary ?*. *Educational Studies in Mathematics, 68(3), 195 - 208.* https://doi.org/10.1007/s10649-007-9110-4