AKSIOMA: Jurnal Program Studi Pendidikan Matematika
Volume 15, No. 2, 2026, 1164-1177

ISSN 2089-8703 (Print)
ISSN 2442-5419 (Online)

DOI: https://doi.org/10.24127/ajpm.v15i2.16273

TRANSITIONAL STRUCTURE OF MATHEMATICAL THINKING FROM STRUCTURAL INTUITIVE TO DEDUCTIVE WITH THE FREEMAN FRAMEWORK

Abdul Aziz[1*], Wayan Rumite[2], Sri Suryanti[3], Agung Setiawan[4]

[1.4]Universitas Muhammadiyah Semarang, Semarang, Indonesia
[2]Universitas Negeri Makassar, Makassar, Indonesia
[3]Universitas Negeri Surabaya, Makassar, Indonesia
**Corresponding author: abdulaziz@unimus.ac.id*

Received 17 April 2026; Revised 10 June 2026; Accepted 25 June 2026

Abstract

Formal thinking requires analytical skills to evaluate whether a statement is true or false. However, researchers have found that some students complete the mathematical proof process using intuitive thinking. Students who studied mathematical proof in their early semesters often struggle to construct structural proofs intuitively. Therefore, this study aims to explore students' thought processes in conducting mathematical proofs, transitioning from intuitive reasoning to deductive reasoning, and achieving correct conclusions using Freeman framework. The research employs a qualitative descriptive method, validated by experts in mathematical proof and reasoning, all of whom have at least ten years of teaching experience in the field. The subjects of this study were two students selected based on their thought processes, ranging from intuitive to deductive reasoning, with both providing correct and incorrect answers. Students who struggle with this transition tend to rely on institutional and evaluative assurances. In contrast, those who successfully navigate the transition are able to integrate evaluative, empirical, and a priori warrants into their reasoning. This research suggests that all parties involved in mathematical proof need to pay careful attention to each stage of the reasoning process to ensure accuracy.

Keywords:, Deductive; Freeman Framework; Structural-Intuitive.

Abstrak

Proses berfikir formal membutuhkan keterampilan analitis untuk menunjukkan apakah suatu pernyataan benar atau salah. Akan tetapi, peneliti masih menemukan mahasiswa yang menyelesaikan proses pembuktian matematika melalui pemikiran intuitif. Mahasiswa yang telah mempelajari pembuktian matematis di semester awal, tidak dapat menyelesaikan bukti struktural secara intuitif. Oleh karena itu, penelitian ini bertujuan untuk mengeksplorasi proses berpikir mahasiswa dalam melakukan pembuktian matematis dari struktural intuitif sampai deduktif dengan menghasilkan kesimpulan yang benar menggunakan Freeman Framework. Metode penelitian ini menggunakan metode deskriptif kualitatif. Instrumen penelitian yang digunakan telah melalui tahap validasi oleh para ahli yang memiliki kemampuan di bidang pembuktian matematika dan penalaran matematika. Hal ini didasarkan atas kriteria memiliki pengalaman mengajar pembuktian dan penalaran matematika lebih dari sepuluh tahun. Subjek yang digunakan dalam penelitian ini adalah dua mahasiswa yang dipilih berdasarkan proses pengelompokan berpikir dari intuitif ke deduktif dengan jawaban salah dan benar. Mahasiswa yang melakukan transisi secara tidak tepat bergantung pada jaminan institusional dan evaluatif. Sebaliknya, mereka yang melakukan transisi secara berhasil mengintegrasikan jaminan evaluatif, empiris, dan a priori ke dalam penalaran mereka. Implikasi dari penelitian ini semua pihak yang terlibat dalam pembuktian matematika perlu memperhatikan setiap tahapan dilalui dengan tepat.

***Kata kunci**: Deduktif; Freeman; Struktural intuitif.*

AKSIOMA: Jurnal Program Studi Pendidikan Matematika ISSN 2089-8703 (Print)
Volume 15, No. 2, 2026, 1164-1177 ISSN 2442-5419 (Online)

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INTRODUCTION

Effective thinking is essential for strengthening conjectures and establishing solid algebraic proofs (Shongwe & Mudaly, 2021; Kiliçaslan & İskenderoğlu, 2023; Akkurt & Durmuş, 2022) and it is important part of the mathematical proving process, because students can use logic in the form if P then Q, which appears in the thinking process (Kosko & Singh, 2019). Mathematical proof convinces others of a claim's validity. It relies on students' reasoning, logic, concepts, and structured problem-solving (Imamoglu & Yontar Togrol, 2021). Arguments aside, there are several components that can be used to convince someone. The investigation of mathematical proving arguments can be used in Toulmin's scheme. One of the components of the Toulmin'scheme is warrant, which is identified with the Freeman framework which consists of a priori, empirical, institutional and evaluative.

Warrants play an important role in drawing conclusions and reasoning in an argument because the truth of a claim can be identified through a warrant, which suggests that disclaimers and qualifications should be eliminated in mathematical proofs. Therefore, the focus is on warrants and backing as guarantors to reach conclusions. In addition to warrants and backing in a mathematical argument, there are rebuttals and qualifiers. A rebuttal is a different statement in conveying an argument, while a qualifier is a belief in an argument.

Warrants are guarantors that can be used to draw conclusions from mathematical proofs (Dahlan et al., 2024). There are three types of warrants in mathematics, namely deductive warrants, structural- intuitive thinking and inductive warrants. Deductive warrants are used to derive correct conclusions based on axioms, algebraic manipulation and counterexample. Structural-intuitive thinking are used to observe experience and mental structures in arriving at a conclusion. Inductive warrants are used to ensure the truth of conjectures made by students based on specific statements leading to general statements (Dahlan et al., 2024).

Students studying algebraic structure courses in tertiary institutions should understand the concept of groups. They need to know the axioms required for a set or set of numbers with binary operations in order to prove groups. In line with what was conveyed by Zetriuslita et al., (2017) that the concept of numbers can train students' thinking skills in learning mathematical proof.

The student mistakes in proving groups for certain sets of numbers occur intuitively because they do not prove it thoroughly and are not aware (Sanwidi, 2018; Setiawan, 2022). This is related to thinking skills that these skills are very important in elementary schools, middle schools and even at the tertiary level (Kurniansyah et al., 2022). College level students should be able to explain mathematical proofs related to algebraic structures correctly based on the mental structures they do. According to Piaget, a person can carry out formal or abstract thought processes after 12 years or more. The changeover of thinking processes, students can do formal thinking to solve mathematical problems or prove mathematically (Putra et al., 2024).

Although Tall and Piaget's theory is related to thinking skills, these concepts cannot be used in understanding students' thinking processes in terms of abstractions that make

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justification in proving mathematical problems. Therefore it is necessary to have a Masonic framework to observe students' thinking processes related to mental structures. The Mason Framework effectively captures the intricacies of the thinking process, while the Toulmin Model provides a robust structure for argumentation. Additionally, the Freeman Framework offers a clear classification of warrants. Together, these three aspects constitute the foundational constructs of this research, ensuring a comprehensive analysis.

The attack stage is marked by the emergence of allegations that show confidence in the claim. The review stage is marked by student activity in checking the accuracy of the completion results, completion in accordance with the questions, giving reasons to guarantee the right solution and the implications of the arguments. Thinking mathematically is a way of learning mathematics, because students can construct conjectures by exploring problems, formulating arguments, justifying arguments, and proving arguments (Puspa et al., 2020). Arguments are defined as statements that need to be verified (Faizah et al., 2020).

Some researchers verify the teacher's mathematical arguments using the Toulmin scheme through several warrants, and some of them focus on analyzing students' arguments in solving mathematical proof problems based on the Toulmin scheme (Metaxas et al., 2016; Tristanti et al., 2017; Laamena et al., 2018). Gholami, (2023) also examined students' thinking processes in solving math problems. Experts research Toulmin's theory and Mason's framework. These experts focus on two warrants in discovering students' arguments: structural-intuitive thinking (Faizah et al., 2020) and the construction of deductive warrants from inductive warrants (Tristanti et al., 2016).

Previous researchers have examined Toulmin and Mason's theory separately, and no one has examined structural-intuitive thinking as a basis for constructing deductive warrants. The deductive warrants may originate from structural-intuitive thinking. Therefore, this study aims to analyze students' thought processes in mathematical proofs, focusing on how intuitive structural-intuitive thinking leads to deductive reasoning.

METHODS

This research is a type of exploratory research with a qualitative approach. The reason for using this approach is to analyze students' thinking skills which require qualitative data in expressing mathematical evidence orally and in writing. Descriptive information in writing is in accordance with the statement of As'ari et al., (2019) that qualitative research can use qualitative descriptive methods. The research design is classified as exploratory as Adams et al., (2016) stated that thinking is a form of communication between a person and himself that takes place in the brain, so that other people cannot access it.

This research was carried out at a prominent university in Semarang, the vibrant capital of Central Java Province. The participants were mathematics education students who had successfully completed a course on algebraic structures. Among the 44 students, 29 engaged with the Masonic framework for their proofs, which were classified into two distinct categories. The first category showcased students who presented mathematical proofs in innovative ways, reflecting their

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Volume 15, No. 2, 2026, 1164-1177 ISSN 2442-5419 (Online)

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distinctive thought processes during problem-solving. The second category included students whose methods were more conventional and widely recognized in the context of problem resolution. From these groups, one student was carefully selected to serve as a research subject: M1 exemplified answers that were unique and seldom seen, while M2 embodied approaches that were more commonly encountered in problem-solving. The selection of students for this research was intentional they demonstrated the ability to address mathematical proof challenges intuitively, relying on conjecture, while effectively conveying the knowledge they possessed. This study not only highlights the diversity of thought in mathematical reasoning but also emphasizes the importance of understanding different problem-solving strategies within the educational context. The relationship between conjecture and proof is the most important thing to identify students' understanding of a theorem or mathematical statement (Zayyadi et al., 2020).

Data collection in this study was carried out by looking at students' thinking processes through the results of test work and interviews. Validity and reliability in qualitative research are carried out through triangulation (Utami et al., 2018). Two qualified validators rigorously evaluated the research instrument, ensuring its effectiveness for both the questionnaire and the structured and semi-structured interviews. Their insightful feedback led to significant improvements in the answer format, making it more systematically organized around mathematical sequences. Thanks to this thorough validation process, the interview instrument is now not only validated but also primed for impactful use in research. The triangulation method applied in this study effectively compared data from both test results and participant interviews, ensuring a robust and comprehensive analysis for each individual involved. The proof of the test used is an introductory question on the algebraic structure of group material. The test is used as a research instrument which is indicated as follows:

Solve the proof problems below:
1. Operation $*$ on $\mathbb{Z}^+$ is defined as $a * b = \max(a, b) + 3$
   Prove that $(\mathbb{Z}^+, *)$ is closed
2. Prove that the set of integer multiples of 4 is a group under the addition operation.

Based on the results of tests and interviews were analyzed using qualitative data. Data analysis was carried out by looking at the results of the subject's written job test and interview transcripts. The subject's thinking process in proving mathematics can be identified from the words conveyed by the subject during think aloud and interviews. Structural-intuitive thinking are used by subjects in identifying problems that are used as a basis for making conjectures and justifications. Conjectures and justifications formed by the subject occur intuitively because the subject identifies problems without going through reasoning and intellectual processes, therefore the guarantees that arise occur quickly and automatically based on prior knowledge. In this research, what is observed is the mental activity carried out by the subject during the process of thinking intuitively with Mason's framework and Toulmin's theory which leads to deductive reasoning. Researchers conducted interviews with subjects related to thought processes to

ISSN 2089-8703 (Print)
ISSN 2442-5419 (Online)

DOI: https://doi.org/10.24127/ajpm.v15i2.16273

reveal in depth the mental activities carried out by the subjects.

RESULTS AND DISCUSSION

Based on the 44 students studied, 29 subjects used Mason's framework and based on the 44 students studied, 29 subjects used Mason's framework and there were 2 students who had specific differences compared to other students in carrying out mathematical proofs intuitively and leading to deductive warrants. The selection of subjects in this study was based on the communication skills possessed by each subject to determine the mental activity carried out during the proving process. The 2 students are M1 and M2. Both perform mathematical proofs in algebraic structures using intuitive thinking which lead to deductive warrants.

Structural-Intuitive thinking are guarantors used by the two students in obtaining conclusions from a proof but these warrants appear intuitively. Structural-Intuitive thinking can be grouped into a priori warrants, empirical warrants, institutional warrants and evaluative warrants(Freeman, 2005). In this study, the premise that there is an structural-intuitive thinking occurs based on the cognitive meaning that appears intuitively(Faizah et al., 2020). The process of forming structural-intuitive thinking can be observed based on the thought process as follows:

First Subject (M1)

M1 proves problem number 1 by identifying based on available information on the problem to be proven. M1 identifies problems by making assumptions based on previously obtained information. According to M1 that $(\mathbb{Z}^+, *)$ is closed without doing reasoning and proof beforehand. Then he proves to check the truth that has been made.

R : Do you think that $(\mathbb{Z}^+, *)$ is closed?
M1 : Yes sir
R : Are you sure about your answer?
M1 : Right sir
R : Have you proved that $(\mathbb{Z}^+, *)$ is closed?
M1 : Not yet proven sir

Based on the results of the proof, this shows that $(\mathbb{Z}^+, *)$ is closed which is made by the subject using the operation according to the problem. The conclusion is correct but there are parts of the work that are not right. M1 made this conclusion unconsciously and it has been proven that $(\mathbb{Z}^+, *)$ is closed, while the solution to $\max(a, b)$is not well understood.

R : What results did you get? is it closed?
M1 : Yes sir
R : Do you understand if the set is closed?
M1 : Yes sir. Three is a member of the set of integers, but I don't understand what $\max(a, b)$ means.
R : $\text{Max}(a, b)$means that from the number a or b, the number with the largest value is taken and then added to 3.
M1 : I see. Okay sir
R : If you replace $\max(a, b)$with the number you have determined, is it closed.
M1 : Closed sir. I take max(2,3) to mean 3 + 3 = 6. 6 is a member of the set of integers
R : What integer, there is a sign like this?
M1 : Positive integer sir

For category question number 2, M1 identifies the problem by explaining the definition of a group which consists

AKSIOMA: Jurnal Program Studi Pendidikan Matematika ISSN 2089-8703 (Print)
Volume 15, No. 2, 2026, 1164-1177 ISSN 2442-5419 (Online)

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of having closed, associative and inverse properties and contains an identity element. Then he arranges multiples of 4 consisting of (4, 8, 12, 16). M1 uses the attack step to prove that it is closed, associative, has an identity element and is inverse. Subject proves closed nature with cayley table but to no avail. In the associative proof stage, the subject can provide correct proof arguments with the explanation (4+8) + 12 = 4 + (8+12).

M1 can determine the identity element well but has difficulty in determining the inverse element. Intuitively what is meant by the identity element is 0 and the example used in the proof to show the identity element is 4 + 0 = 0 + 4 = 4. The difficulty encountered is when proving the inverse element. He is still confused whether the number -4 is a multiple of 4 to denote the inverse element. Therefore he stated that if a number is a multiple of 4 to the addition operation it is not a group. Next, the researcher conducted interviews to find out M1's beliefs based on the thought processes that were carried out.

R : Are you sure about the number 2 group?
M1 : No sir, because there is no inverse element
R : Does it contain elements of identity
M1 : Yes sir. In my mind 4 + 0 = 0 + 4 = 4. So there is an element of identity. The identity element is 0
R : Fine. What if will show the inverse ?
M1 : In my opinion (-4) + 4 = 4 + (-4) = 0. But i'm not sure if (-4) is included in a multiple of 4
R : Why do you say that, if (-4) is not a multiple of 4?
M1 : As far as i know multiples are positive, sir
R : Do you think there is no group for question number 2?
M1 : No sir

Based on the verification of the results of work on number 1 and number 2 by M1 using intuition based on previous knowledge. The warrants used are to prove closed and group characteristics for proving question number 2. The warrants are used to solve both problems intuitively. Therefore, the conclusion is wrong, this error is because in question number 1 they still do not understand the meaning of maximum and minimum, even though at the end they understand the problem. Whereas for question number 2 M1 uses multiples of 4 (4,8,12,16) to indicate multiples of 4. However, multiples of 4 are not only limited to (4,8,12 and 16). M1 was unable to determine the inverse element at verification. Therefore the structural-intuitive thinking of M1 can be illustrated based on Figure 1.

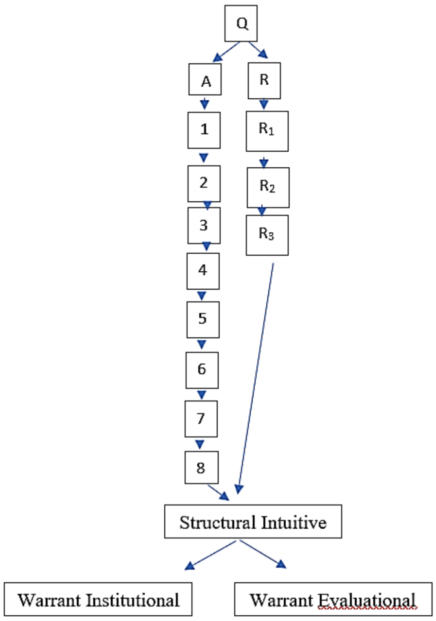

Figure 1. M1's Structural-Intuitive Thinking

AKSIOMA: Jurnal Program Studi Pendidikan Matematika ISSN 2089-8703 (Print)
Volume 15, No. 2, 2026, 1164-1177 ISSN 2442-5419 (Online)

DOI: https://doi.org/10.24127/ajpm.v15i2.16273

Table 1. Intuitive Structural Mindset

Q	Proven question
A	Attacks
R	Reviews
1	use the group definition for proof number 1 and number 2
2	makes the conjecture of ($\mathbb{Z}^{+}$, *) closed
3	don't understand the meaning of max(a,b) in binary operation
4	prove multiples of 4 (4,8,12,16)
5	based on the results of evidence it can be shown (4,8,12,16) to be closed, associative and have an element of identity
6	find it difficult and have doubts in determining whether -4 is an inverse element
7	the answer that is proven from the results of the proof is not quite right because it defines multiples of 4 not as a group
R1	Concluding that question number 1 is closed and number 2 is not a group
R2	justify the correct work steps after checking and reviewing
R3	cannot justify question number 2 because it cannot prove that the inverse element is in accordance with the correct steps

According to the diagram in Figure 1 and the details provided in Table 1 the thought process demonstrated by M1 in carrying out mathematical proofs for algebraic structure problems, structural-intuitive thinking are produced in the form of institutional warrants and evaluative warrants. This type of structural-intuitive thinking appears because he cannot prove based on actual definitions, but only uses the basic knowledge he has. M1 also cannot use the definition of multiplication well so that it can be stated that the flow of thinking in structural-intuitive thinking is unable to lead to inductive reasoning.

Second Subject (M2)

M2 proves questions 1 and 2 by entering the entry stage, namely identifying the problem. Identification of the problem that was carried out to work on question number 1 was conveyed verbally and written as follows:

M2	:	For questions ($\mathbb{Z}^{+}$, *) clearly closed sir
R	:	Why are you sure like that?
M2	:	Every natural number that is added will produce a natural number
R	:	What does max mean here?
M2	:	One moment sir, I will check As far as I know, max shows the maximum
R	:	Well when did you conclude?
M2	:	$\mathbb{Z}^{+}$ closed sir

The results of think aloud and interviews show the identification of the M2 problem against the set $\mathbb{Z}^{+}$ which is operated with (*). After doing the maximum proof and definition of the problem, it can be concluded that the set of natural numbers is closed. For question number 2 M2 shows the entry section to identify the problem using the multiplication definition and group definition.

He explained the concept of multiplication to be $4k, k \in \mathbb{Z}$, so it is closed and associative and is able to show identity and inverse elements so that groups apply. The first step taken regarding the proof with the closed property is that it assumes that $k, l \in \mathbb{Z}$ then $x = 4k$ and $y = 4l$ for each k and l are clearly members of the set of integers. With the addition operation it is clear that k and l are members of the set of numbers $\mathbb{Z}$. The second step to prove associative is obtained $(x + y) + z = x + (y + z)$.

ISSN 2089-8703 (Print)
ISSN 2442-5419 (Online)

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R : How do you determine which multiples of 4 are 4m and (-4) ∈ $\mathbb{Z}$

M2 : From the practice questions that I have done, sir

R : What kind of questions did you do?

M2 : Based on what I know, multiples of 4 can be defined as 4m where m is a member of the set of integers

R : How are you sure if a multiple of 4 is a group?

M2 : Because it fulfills the closed, associative, identity elements and inverse elements

Analysis of the M2 proof results indicates that a structural-intuitive thinking pattern enhances deductive reasoning and can be illustrated based on Figure 2.

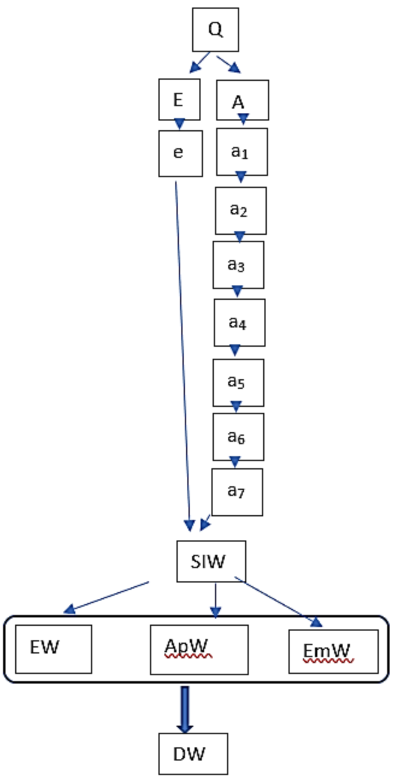

Figure 2. M2's Structural-Intuitive Thinking

Table 2. Structural-Intuitive Thinking Mindset Towards Deductive Warrants

Q	Proven question
E	Entry
A	Attacks
e	Using warrants in the form of definitions of multiples of 4 and definitions of groups that include closed, associative, identity elements and contain inverse elements
a_1	Assuming that $\mathbb{Z}^+$ is closed with operations $max(a, b)$ +3 the result is a member of $\mathbb{Z}^+$
a_2	Proving problem number 1 by using a closed definition
a_3	Proving problem number 2 by changing the definition of multiples of 4 to 4k, k members of $\mathbb{Z}$
a_4	Proving closed properties
a_5	Proving associative properties
a_6	Defines an identity element
a_7	Defines the inverse element
SIW	Structural-Intuitive Thinking
EW	Evaluative Warrant
ApW	A priori Warrant
EmW	Empirical Warrants
DW	Deductive Warrant

According to the diagram in Figure 2 and the details provided in Table 2 the stages of proof carried out by M2, it is known that structural-intuitive thinking to deductive warrants are formed based on Freeman's theory which includes thinking processes that are carried out evaluatively, a priori and empirically. The evaluative stage arises because of the process of evaluating the knowledge or experience one has in doing the proof. At this stage M2 rechecks or evaluates the assumptions made in working on $\mathbb{Z}^+$ questions which are closed and group multiples of 4 apply.

AKSIOMA: Jurnal Program Studi Pendidikan Matematika ISSN 2089-8703 (Print)
Volume 15, No. 2, 2026, 1164-1177 ISSN 2442-5419 (Online)

DOI: https://doi.org/10.24127/ajpm.v15i2.16273

Based on this, an a priori process was carried out because he carried out a proof using pre-existing definitions, namely in identifying the definition of multiples and the definition of groups. Then from the definition that is owned, it is applied to the problem to be proven so that it gives rise to empirical guarantees as a priori real form. Based on the exploration results in this study, new findings can be obtained, namely the process of structural-intuitive thingking to deductive warrants. The process of structural intuitive thinking leading to successful deductive warrants can be seen in Table 3.

Table 3. Formation of Structural-Intuitive Thinking Towards Deductive Warrants Based on Freeman's Theory

Aspect	**M1 (Incorrect Proof)**	**M2 (Correct Proof)**
Warrants Used	Institutional, Evaluative	Evaluative, Empirical, A priori
Main Error	Misinterpretation of max (a, b), incorrect inverse element	Successfully applied group axioms
Outcome	Unable to transition to deductive reasoning	Transitioned successfully to deductive proof

Based on Table 3 this study shows that the thinking processes used by students in solving mathematical proofs related to algebraic structure problems produce structural-intuitive thinking warrants in the form of institutional warrants and evaluative warrants because the first subject performs proof in the form of numbers. The subject did the proof inaccurately because he did not understand the meaning of "max" in the question and changed the multiples of 4 to (4,8,12,16) while the integer multiples of 4 were not limited to that, so the subject did the proof incorrectly in proving the group axioms. Institutional warrants are not formed based on existing facts but are formed based on internal findings or appear externally based on experience. Furthermore, evaluative warrants are a process of evaluating knowledge in conducting evidence, so the truth still needs to be proven. In this finding, the first subject is only able to think structurally intuitively and has not been able to lead to deductive thinking because in the proof there is no deductive thinking process found which includes axiom deduction, algebraic manipulation or the use of counterexample.

The second subject in doing structurally intuitive evidence includes evaluative warrants, empirical warrants and a priori warrants. The evaluative warrant is the same as the definition above, namely how the subject will evaluate the experience he has in proving. Whereas the a priori warrant appears intuitively based on previously held concepts related to sets of multiples of 4 and also the axioms in the definition of groups. The second subject used the definition of a multiple of four to be $\{4k, k \in \mathbb{Z}\}$. The second subject uses algebraic manipulation, proving closed, associative, identity and inverse elements one by one. Empirical warrants are warrants that appear intuitively based on the facts contained in a priori definitions. The second subject applied the correct definition in performing an algebraic proof.

Structural-intuitive warrants are obtained from the thinking processes carried out by each subject based on Mason's framework of Entry, Attack

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Volume 15, No. 2, 2026, 1164-1177 ISSN 2442-5419 (Online)

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and Review and linked to Toulmin's theory which includes Data, Claims, Warrants, Backing, Rebuttal and Qualifier. Each subject requires a thought process based on mental activity carried out intuitively, as a result the conclusions made have two possibilities, which can be right and wrong. This mental activity is in the form of cognitive processes carried out to solve mathematical problems in this case mathematical proof. The cognitive activity analysis carried out can be identified from the difficulties experienced, the reasoning process carried out, the determination of completion strategies, and the evaluations carried out (Giacomone et al., 2019). The process of thinking that occurs automatically, without going through rational and intellectual reasoning, unconsciousness or lack of effort in solving mathematical problems is called intuitive thinking. The warrants used by students in carrying out mathematical proofs through thinking intuitively can occur implicitly and explicitly.

The results of this study indicate that structural-intuitive insights leading to deductive reasoning can occur because of the mental activity carried out by students in the form of thinking processes that occur in the brain which then lead to deductive thinking (Rumite et al., 2026). According to Freeman, the mental activities carried out by students produce structural-intuitive warrants in the form of a priori warrants, empirical warrants, institutional warrants, and evaluative warrants. The four warrants are guarantors used by students in proving related to algebraic structural problems that arise intuitively. Evaluative warrants are the process of evaluating the method of proof that is done based on experience. A priori warrants are warrants used by students in conducting mathematical proofs in the form of definitions and axioms. Empirical warrants can be interpreted as the application of a priori existing definitions in mathematical proofs. Utilizing the APOS theory to investigate the shortcomings of M1 uncovers significant deficiencies, especially in the process (Rosyidi & Hasanah, 2022). A breakdown in the internal understanding process can severely hinder the object and schema processes, ultimately affecting the results (Listiawati et al., 2025; Nurlaelah et al., 2025). Conversely, M2 demonstrates success as each of these processes operates seamlessly, producing outcomes that align perfectly with the established standards (Leton et al., 2020; Noto et al., 2016). This contrast highlights the critical importance of a robust internal understanding for achieving optimal results.

The process of structural-intuitive thinking warrants leading to deductive warrants contains three aspects of structural-intuitive thinking, namely evaluative warrants, a priori warrants and empirical warrants. After going through these three warrants students are able to perform algebraic manipulation as shown by using the definition of multiple integers to be $\{4k, k \in \mathbb{Z}\}$. Algebraic manipulation is one indicator of deductive warrants (Jatisunda et al., 2025). Unlike the case with the first subject. He is only capable of executing structural-intuitive thinking warrants which contain evaluative warrants and institutional warrants and is unable to use algebraic manipulation so that structural-intuitive thinking are unable to proceed to deductive warrants.

The thinking process carried out by students while doing mathematical proof occurs through identifying problems, making conjectures, testing the truth of conjectures by describing

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evidence one by one, making conclusions and re-examining the evidence that has been produced to make justifications. This process is an illustration in the proof that is carried out from an intuitive process to a deductive one by including evaluative warrants, apriori warrants and empirical warrants. He was only able to execute the warrant of structural-intuitive thinking containing evaluative warrant and institutional warrant and was unable to use algebraic manipulation so that structural-intuitive thinking was unable to continue to deductive warrant.

The thinking process carried out by students when conducting mathematical proof occurs through identifying problems, making conjectures, testing the truth of conjectures by describing evidence one by one, making conclusions and re-examining the evidence that has been produced to conduct justification. This process is a description of the proof carried out from the intuitive process to the deductive process by including evaluative warrants, a priori warrants and empirical warrants. Teachers need to provide more assistance to students in teaching algebraic manipulation with various proof methods. Further research can be directed to determine the structural intuitive thinking scheme to inductive and identify the differences that arise with the results of this study.

CONCLUSIONS

This study aimed to explore students' thinking processes in mathematical proof, particularly the transition from structural-intuitive thinking to deductive reasoning using the Freeman framework. The findings reveal that students employ different types of warrants during the proving process, which significantly influence the correctness of their conclusions. Students who rely primarily on institutional and evaluative warrants tend to produce incomplete or incorrect proofs due to limited understanding of definitions and lack of algebraic manipulation.

This condition prevents them from progressing toward deductive reasoning. In contrast, students who successfully transition to deductive reasoning integrate evaluative, empirical, and a priori warrants. These students are able to apply definitions, perform algebraic manipulation, and systematically verify group properties, leading to valid mathematical proofs.

Furthermore, the study confirms that structural-intuitive thinking can serve as a foundation for deductive reasoning when supported by appropriate conceptual understanding and reflective evaluation. The thinking process follows a sequence of identifying problems, forming conjectures, testing them through logical steps, and re-evaluating the results. Therefore, the transition from intuitive to deductive thinking is not automatic but requires the integration of multiple types of warrants and well-developed mathematical reasoning skills.

Based on the findings of this study, several recommendations can be proposed. First, lecturers should provide more structured guidance in teaching mathematical proof, especially in emphasizing the correct use of definitions, axioms, and algebraic manipulation. Instruction should explicitly bridge intuitive understanding and formal deductive reasoning. Second, learning activities should be designed to train students in evaluating their own reasoning processes, such as through reflective tasks, think-aloud strategies, and guided proof construction.

DOI: https://doi.org/10.24127/ajpm.v15i2.16273

This can help students strengthen evaluative, empirical, and a priori warrants in their thinking. Third, future research is recommended to further investigate the development of structural-intuitive thinking toward other forms of reasoning, such as inductive reasoning, and to examine differences across larger and more diverse subject groups. Moreover, establishing instructional models or learning frameworks that effectively bridge the gap between intuitive and deductive thinking would greatly enhance students' proofreading skills, leading to higher academic success.

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ISSN 2089-8703 (Print)
ISSN 2442-5419 (Online)

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