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STUDENT STRATEGIES AND ERRORS IN SOLVING MATHEMATICAL REPRESENTATION PROBLEMS ON A QUADRATIC FUNCTION MATERIAL

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Abstract

Students often experience difficulties in using multiple mathematical representations when solving quadratic function problems, particularly in connecting visual, symbolic, and verbal forms. While previous studies have mainly focused on identifying students' errors, the strategies underlying these errors remain insufficiently explored. This study aims to identify the students' strategies when solving mathematical representation problems on quadratic functions material. Using a descriptive qualitative approach, written responses from 127 students at two public high schools in Pekanbaru were analyzed and supported by interviews with six selected students to clarify their reasoning. The findings show that students predominantly relied on procedural strategies, such as point substitution or mechanical formula use, with limited conceptual understanding. Although most students attempted the visual task, many produced inaccurate graphs. The symbolic and verbal indicators showed very high non-response rates, indicating difficulties in translating contextual information into algebraic form and expressing mathematical reasoning. These results highlight the need for instructional approaches that integrate multiple representations and strengthen conceptual connections.

Keywords: Errors; mathematical representation; quadratic function; strategies.

Abstrak

Banyak siswa masih mengalami kesulitan dalam menggunakan berbagai bentuk representasi matematis ketika menyelesaikan soal fungsi kuadrat, terutama dalam menghubungkan representasi visual, simbolik, dan verbal. Meskipun penelitian sebelumnya lebih banyak berfokus pada identifikasi kesalahan yang dilakukan siswa, strategi yang mendasari munculnya kesalahan tersebut masih belum banyak dikaji. Penelitian ini bertujuan untuk mengidentifikasi strategi yang digunakan siswa dalam menyelesaikan soal berbasis representasi pada fungsi kuadrat serta mengklasifikasikan kesalahan yang muncul pada indikator visual, simbolik, dan verbal. Penelitian ini menggunakan pendekatan kualitatif deskriptif dengan menganalisis jawaban tertulis dari 127 siswa di dua SMA negeri di Pekanbaru, disertai dengan wawancara terhadap enam siswa terpilih untuk memperjelas alasan dan proses berpikir mereka. Hasil penelitian menunjukkan bahwa siswa cenderung menggunakan strategi prosedural, seperti substitusi titik atau penggunaan rumus berdasarkan hafalan, dengan pemahaman konseptual yang terbatas. Meskipun sebagian besar siswa mencoba menyelesaikan soal visual, grafik yang dihasilkan seringkali tidak akurat. Pada indikator simbolik dan verbal, tingkat non-respons sangat tinggi, yang menunjukkan kesulitan siswa dalam menerjemahkan informasi kontekstual ke dalam bentuk aljabar dan dalam mengungkapkan penalaran matematis. Temuan ini menekankan pentingnya pendekatan pembelajaran yang mengintegrasikan berbagai bentuk representasi dan memperkuat hubungan konseptual di antara representasi tersebut.

Kata kunci: *Fungsi kuadrat; kesalahan; strategi; representasi matematis.*

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INTRODUCTION

Quadratic functions are a fundamental topic in secondary school mathematics because they serve as a foundation for understanding nonlinear functions, higher-degree polynomials, function transformations, and later topics such as calculus (Suominen, 2018), (Listiawati & Juniati, 2021). Their algebraic and geometric structures enable students to explore essential mathematical ideas, including the vertex, axis of symmetry, maximum or minimum values, and the relationship between coefficients and the shape of the graph. Mastery of quadratic functions is therefore strategic, functioning not only as core content but also as a conceptual bridge between basic and advanced mathematics.

Learning quadratic functions requires students to understand and coordinate multiple forms of repre-sentation. In mathematics education, representations are defined as signs or combinations of signs, symbols, images, objects, diagrams, or graphs, used to communicate and construct conceptual meaning (Mainali, 2021). Because mathematics is inherently abstract, representations help students access, visualize, and express mathematical ideas more meaningfully (Johnson, 2018). Common forms of representation include verbal, graphical, symbolic, and numerical formats (Mainali, 2021), while other classifications identify five categories: visual, symbolic, verbal, physical, and contextual (Hatisaru, 2020). This study focuses on three central forms, visual, symbolic, and verbal representations, because they are most frequently used in classroom instruction and play a crucial role in developing conceptual understanding of quadratic functions.

Each mathematical topic, including quadratic functions, involves complementary representations. Symbo-lic representation appears in the general form of the quadratic function and in procedures for determining roots. Visual representation is found in the parabola, which illustrates properties such as the vertex, axis of symmetry, and the function's extrema. Verbal representation is seen in word problems or narrative explanations related to the concept. A comprehensive under-standing of quadratic functions there-fore requires students to translate flexibly across these representations depending on the problem context.

Students' ability to express and communicate mathematical ideas to understand problems and develop appropriate solutions is referred to as mathematical representation ability (Husain et al., 2022; Nasrun et al., 2023). Its indicators include visual, symbolic, and verbal representation (Hilsania & Masrukan, 2023). This ability enables students to visualize abstract ideas, communicate mathe-maticcally, and solve problems analy-tically (Hariyani et al., 2023; Mainali, 2021). Without adequate representa-tional skills, students struggle to connect algebraic expressions, graphs, and verbal interpretations, which inhibits conceptual understanding and problem-solving competence.

However, numerous studies show that Indonesian students' representa-tional abilities remain low. Research by Shinariko et al. (2021) reports that overall representation ability is in the low category, with symbolic and verbal representations classified as very low and low. Students frequently have difficulty interpreting symbols, trans-forming verbal information into algebraic forms, and constructing

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accurate graphs. Similar results were found by Rahmah et al. (2022), revealing that students with moderate and low abilities meet only a portion of the representation indicators, particularly verbal, and often fail to construct mathematical expressions or graph sketches consistent with the problem context. Findings from Fitrianna et al. (2018) show that students often ignore key information and rely solely on visual cues, leading to representations that diverge from the intended concept. Collectively, these studies highlight a persistent gap between the representational demands of mathematics instruction and students' actual abilities to integrate visual, symbolic, and verbal forms. These findings indicate that students still experience substantial difficulties in coordinating multiple representations when solving quadratic function problems.

Although previous research on quadratic functions has examined students' errors, most studies have focused on categorizing error types using frameworks such as mathematical literacy (Hidayanti & Hidayati, 2025), analytical thinking (Azizah et al., 2021), or Newman procedures (Pramesti et al., 2024). These studies provide valuable insights into the types of mistakes students make when solving mathematical problems. However, they mainly emphasize the classification of errors and do not sufficiently examine the strategies students employ when working with different forms of representation. While these studies provide insight into common errors, they do not investigate the strategies students use when working with different forms of representation. As a result, the relationship between students' representational strategies and the errors they produce remains insufficiently explored, particularly in the context of quadratic functions involving visual, symbolic, and verbal representations.

Addressing this gap, the present study aims to identify the strategies students use when solving representation-based problems on quadratic functions and to classify the types of errors that emerge within visual, symbolic, and verbal indicators. By focusing on students' strategies, this study provides a deeper explanation of how representational difficulties develop and why errors occur when students attempt to translate information across multiple forms of representation.

This study is expected to contribute theoretically to the literature on mathematical representation and practically to instructional design. From a theoretical perspective, documenting how students use or fail to use representational strategies provides insight into the development of representational flexibility, a critical component of mathematical thinking. From a practical perspective, the findings may inform the design of instructional interventions that emphasize coordinated use of multiple representations, enhance conceptual connections among them, and support students in translating ideas across visual, symbolic, and verbal forms. Such interventions are particularly important in the context of quadratic functions, where understanding the connections among algebraic structure, graphical behavior, and contextual meaning is essential.

METHODS

This study employs a descriptive qualitative approach aimed at identifying problem-solving strategies and types of errors made by students in

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solving mathematical representation problems on the topic of quadratic functions, focusing on three indicators: visual, symbolic, and verbal. This approach was chosen because it allows the researcher to deeply explore students' thought processes, including how they select and use various forms of mathematical representation, as well as how their errors emerge within the context of the strategies employed. The research was conducted through several operational stages, namely instrument development and validation, administration of the mathematical representation test, selection of interview participants based on test results, semi-structured interviews, and qualitative data analysis of students' strategies and errors.

The subjects of this study consist of 127 tenth-grade students from two public senior high schools in Pekanbaru, with 67 students from the first school and 60 students from the second school. The two schools were selected purposively because they provided access for data collection and represented similar academic characteristics among public senior high schools in Pekanbaru. The study was conducted during the even semester of the 2023/2024 academic year after students had received instruction on quadratic functions. Both schools are public institutions that implement the Merdeka Curriculum and had delivered instruction on quadratic functions before the data collection was conducted. The 127 students were selected using purposive sampling, targeting students who had received instruction on quadratic functions and were willing to participate in a mathematical representation ability test. This sampling strategy was used to ensure that participants had sufficient prior exposure to the quadratic function material being investigated in the study and were willing to participate as research subjects.

Data collection was conducted using a mathematical representation ability test and interview guidelines. The research material investigated in this study focused on students' understanding of quadratic functions, particularly concepts related to graphing quadratic functions, determining vertices, interpreting contextual quadratic problems, and explaining relationships between graphical, symbolic, and verbal representations. The test consisted of three questions designed based on the three representation indicators: visual, symbolic, and verbal. The questions were intended to explore how students solved problems using various forms of representation and their ability to connect and transform information across different representations. The interview guidelines, also structured around the same indicators, were used to further explore students' thinking strategies and clarify the types of errors they made.

Data in this study were collected using two techniques: tests and semi-structured interviews. The test was administered during regular mathematics class hours under the direct supervision of the subject teacher. Students completed the test individually within 90 minutes. Prior to the test, the teacher provided a technical explanation of the question format. The teaching conditions at both schools generally followed a conventional approach, in which the teacher remained the dominant source of information and the use of visual aids such as graphs or digital media had not yet become a routine practice. Based on preliminary observations and input from the teachers, students at these schools had

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heterogeneous mathematical abilities, with the majority demonstrating medium to low levels of understanding in algebraic concepts and function graphs.

Semi-structured interviews were conducted with six students who were selected based on specific criteria derived from their written test responses. After the test was scored, students' answers were reviewed to identify cases that required clarification, such as incorrect solutions, incomplete reasoning, or non-response in one or more representation indicators. This criterion-based purposive sampling approach was used to select information-rich cases whose written responses revealed unclear or error-prone strategies and therefore required further probing. The students were chosen because their responses demonstrated substantial difficulties or absence of solutions, making them suitable for clarifying how their strategies led to the errors identified. The semi-structured format allowed the researcher to probe students' reasoning about their written answers, to clarify why they chose certain strategies, and to better understand the sources of their errors in each representation indicator.

Before being administered to students, the research instruments were validated by three experts in mathematics education to ensure the clarity of the questions, the relevance of the items to the representation indicators, and the suitability of the tasks for measuring students' understanding of quadratic functions. This validation process ensured the content validity of the instruments used in the study.

The data were analyzed qualitatively through several stages. The first stage was data reduction, which involved simplifying, organizing, and focusing students' answers to align with the research objectives. At this stage, all student answers were examined, including incomplete, irrelevant, or blank responses. The next stage was the categorization of strategies and errors, by grouping the problem-solving strategies used by students. The identified errors were analyzed and classified into several types, namely conceptual errors, procedural errors, and non-response. Interview data were analyzed to support and clarify the interpretation of students' written responses, particularly in understanding the reasoning processses that led to specific strategies and errors.

The trustworthiness of data in qualitative research is assessed through four main criteria: credibility, transferability, dependability, and confirmability. As explained by Stahl and King (2020), these four criteria originnate from the Lincoln and Guba framework and are essential for enhancing the readers' confidence in the research findings. According to Korstjensa and Moserb (2018), credibility refers to the confidence that the findings truly reflect the partici-pants' data, transferability concerns the extent to which findings can be applied to other contexts, dependability relates to the consistency of the research results over time, and confirmability empha-sizes that the findings are grounded in the data and not influenced by researcher bias.

In this study, credibility was ensured through instrument validation by three mathematics education experts and methodological triangulation between written tests and interviews. Interviews were also used for member checking to ensure that the researcher's interpretations aligned with the students' thoughts. Transferability was supported through the provision of thick descriptions of the schools, participant characteristics, and learning context.

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Dependability was fulfilled through systematic documentation of the entire research process, forming an audit trail. Confirmability was upheld through validation of the analysis results by two mathematics teachers to ensure that the classification of strategies and errors was objective and aligned with actual classroom practices. These two teachers were not directly involved in the analysis, allowing them to independently assess the objectivity of the findings.

RESULTS AND DISCUSSION

This study analyzes the results of a mathematical representation ability test taken by Grade X students on the topic of quadratic functions, based on three main indicators: visual (question number 1), symbolic (question number 2), and verbal (question number 3). The participants consisted of 127 students from two public high schools in Pekanbaru City. The test results indicate that the majority of students experienced difficulties in completing the tasks with the required representations accurately and thoroughly. There was also a notable disparity in understanding across the different forms of representation, with the symbolic indicator showing the highest failure rate (Table 1).

Table 1. Students' performance on representation indicators

Category	Representation Indicator (%)		
	Visual	**Symbolic**	**Verbal**
Correct	3.9	1.6	0
Incorrect/ incomplete	87.4	21.2	41.7
No-response	8.7	77.2	58.3
Total	**100**	**100**	**100**

The distribution of these results reveals that mastery of all three forms of representation remains far from the expected level. Visual representation may appear more procedurally accessible, yet many errors were found in drawing the graph of a quadratic function. Symbolic representation appears to be the most avoided by students, as evidenced by the large number of unanswered responses. Meanwhile, the ability to explain or reflect mathematical information in verbal form remains weak, indicating a lack of integration among the various forms of mathematical representation.

The visual item asked students to draw the graph of the function $f(x) = x^2 - 4x + 8$ and determine its vertex. Most students used a strategy of substituting several x values into the quadratic function to obtain coordinate points for the graph. Commonly used x values included -1, 0, 1, 2, and 3. These points were then connected to form a curve. However, some students did not consider the symmetry of the parabola or calculate the vertex of the curve before selecting the x values for substitution. As a result, the graphs drawn did not resemble a proper parabola. In several cases, the curves appeared as broken lines or even resembled straight lines. These errors occurred because students failed to understand that a quadratic function graph has a specific structure, namely a symmetrical parabola with the vertex as a key element. An example of a student's answer is shown in Figure 1.

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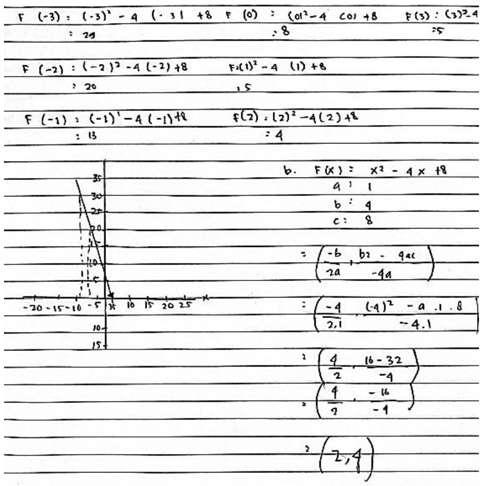

Figure 1. Written response of participant S1

Figure 1 shows the written response of student S1. The student attempted to construct the graph by substituting several x-values such as $-3, -2$, and -1 into the function and then plotting the resulting points. This strategy is procedurally acceptable for graphing a quadratic function. However, the student did not first determine the vertex or consider the axis of symmetry of the parabola. As a result, the selected x-values were not distributed symmetrically around the vertex $x = 2$. Furthermore, the plotted points were connected using straight segments, producing a broken line rather than a smooth parabolic curve. This indicates that the student interpreted the graph as a collection of isolated points rather than as a continuous quadratic relationship. Although the student later attempted to determine the vertex using the formula and obtained the coordinate (2,4), this information was not integrated correctly into the graph. Consequently, the resulting graph does not accurately represent the structure of the quadratic function.

During the interview, the student stated, "I just took a few x values, because I was afraid of doing too many. Then I just plotted the points." This indicates that the solution process was carried out mechanistically, without a deep understanding of the properties of a parabola or the relationships among the coordinate points. Some students also did not mention or label the vertex, even though it was explicitly asked in the problem. Furthermore, a few students who did mention the vertex made errors in writing its coordinates, either because the graph was incorrect or because they did not know the formula for the vertex.

Next, the symbolic item was based on the context of the Angry Birds game, in which students were asked to determine when the bird would hit the ground, the initial height of the launch, and the maximum height, based on the quadratic function $h(t) = -3t^2 + 6t + 45$. Conceptually, solving this problem required students to translate verbal information into symbolic procedures, such as finding the roots of the quadratic function and determining the maximum point of the parabola.

However, out of the 127 students, 98 chose not to answer. Among those who did respond, many did not know where to begin. Most students were unable to transform the story context into an algebraic form or an analyzable quadratic function. The strategies used by students varied widely, ranging from attempting to find the maximum point using formulas, applying the discriminant to find the roots, to guessing values based on the illustration. However, most did not complete the process. A small number attempted to use the quadratic formula, but still made errors in substitution or in calculating the discriminant. An example of a student's answer is shown in Figure 2.

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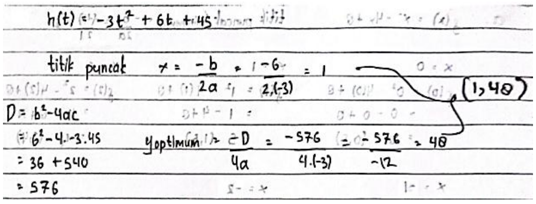

Figure 2. Written response of participant S2

Figure 2 shows the written response of student S2. The student began by identifying the quadratic function $h(t) = -3t^2 + 6t + 45$ and attempted to determine the maximum point of the parabola. The student correctly applied the vertex formula $t = -\frac{b}{2a}$ and obtained $t = 1$. The student then calculated the maximum height using the discriminant-based formula $y_{optimum} = -\frac{D}{4a}$ and obtained the value 48. Thus, the student correctly identified the vertex of the function as $(1,48)$, which represents the maximum height of the bird.

However, the student did not answer the other parts of the problem. The task required determining when the bird hits the ground and the initial launch height, but these were not addressed. To determine when the bird hits the ground, the student should have solved the equation $h(t) = 0$ to find the roots of the quadratic function. Additionally, the initial height could be obtained directly from $h(0) = 45$. The student's work indicates that the solution focused only on applying a memorized procedure for finding the vertex, without fully interpreting the contextual meaning of the problem. This suggests that the student's symbolic reasoning was procedural rather than conceptual, and that the connection between the quadratic model and the physical context of the problem was not fully understood.

During the interview, the student explained, "Since the question asked for the maximum height, I remembered x and y optimum. As for the question about when the Angry Bird hits the ground and from what height it was launched, I didn't know what formula to use, so I didn't answer." This student did not understand the difference between a maximum value and a maximum point, thus the response was given as coordinate points. Additionally, the student could only interpret story problems if the terminology used matched that of mathematical terms. This indicates that the student's symbolic understanding was still unstable and that translational skills from story problems to equations were still low. The student was not yet able to build a relationship between time and height information and the appropriate mathematical form. Furthermore, responses from other students revealed procedural difficulties and misconceptions related to the signs of coefficients.

In the verbal indicator, students were asked to compare the maximum points of two quadratic functions presented in graphical and symbolic forms. This question required students to link visual and symbolic representations and express them in a logical and clear verbal argument. However, 74 students left the question unanswered, and the remaining 53 gave very brief responses that lacked mathematical reasoning.

Some students attempted to explicitly determine the vertex coordinates of each function. However, many were unable to explain the steps of their solution in a coherent verbal explanation. Students' written sentences tended to be incomplete, illogical, or irrelevant to the question. Additionally,

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misconceptions were found among students, as some did not know the difference between the maximum value and the maximum point. An example of a student's answer is shown in Figure 3.

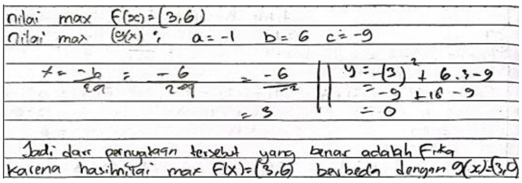

Figure 3. Written Response of Participant S3

From the student's response in Figure 3, it is evident that there was a misconception between the maximum value and the maximum point. In the written solution, the student conclude (translated from Indonesian), "So, based on these statements, the one who is correct is Fika because the maximum value of $f(x) = (3,6)$, which is different from $g(x) = (3,0)$". This shows that the student treated the ordered pair $(x, f(x))$ as the "maximum value" of the function. During the interview, the student stated, "I think maximum value means the same as maximum point." Several other students simply wrote, "Fika is correct" or "Candra is correct" without supporting their answers with reasons, such as calculating the maximum value or analyzing the vertex. Some others gave inappropriate reasons in their answer sheets, such as, "because one uses a graph and the other uses a formula, so the maximum values are different." Another interviewed student said, "If it's already in graph form, I get confused about how to calculate it. I understand better when there's a formula." One student also expressed, "I know the answer, but I get confused when I have to write it down. I'm afraid of giving the wrong reason." This indicates a lack of practice in integrating and comparing information across representations, as well as weak verbal mathematical communication skills.

Based on the classification of findings from 127 students, the errors encountered can be grouped into four main categories: conceptual errors, procedural errors, representational errors, and non-response. A detailed presentation is shown in Table 2.

Table 2. Distribution of student errors on each representation indicator

Indicator	Conceptual	Procedural	Non-response
Visual	65	46	11
Symbolic	7	20	98
Verbal	18	35	74

In the visual indicator, conceptual errors were the most dominant, with 65 students failing to understand that the graph of a quadratic function is a parabola and that there is a relationship between the coefficients of the quadratic function and the shape of the parabola. In addition, students did not understand that the graph of a quadratic function is symmetrical with respect to the vertical axis. This misunderstanding led students to draw graphs as straight lines or to omit identifying the vertex type. This error is supported by student interviews, one of which stated, "To draw a graph, I usually substitute several points, including negative numbers, zero, and positive numbers, then I connect the resulting points with a straight line". A total of 46 students made procedural errors, such as incorrect substitution of x values or mistakes in plotting coordinate points. Meanwhile, 11 students did not provide any response (non-response).

In the symbolic indicator, 98 students did not answer or merely copied the question without attempting a solution, which is categorized as non-

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response. Among the 29 students who attempted to answer, 20 procedural errors were identified, such as mistakes in substitution, factoring, or discriminant calculation. Conceptual errors were found in 7 students, stemming from a lack of understanding of the relationship between algebraic forms and the context of the problem. Interviews revealed that some students were unaware of the connection between the question and the concept of quadratic functions, such as understanding that determining when Angry Bird hits the ground means finding the zero of the function.

In the verbal indicator, 18 students made conceptual errors, for example, by not understanding the meaning of the vertex or by concluding the correctness of an answer based only on subjective opinion without mathematical justification. Meanwhile, 35 students made procedural errors, such as answering only parts of the question without explanation, providing results without including the characteristics of the function, or giving general reasoning without constructing a proper verbal argument. One student stated, "I know the answer, but I'm confused about how to write it. I'm afraid of giving the wrong explanation”. In addition, 74 students did not provide any answer or merely copied the question, thus falling into the non-response category. The high rate of non-response indicates significant difficulty in transforming graphical and symbolic understanding into clear and coherent verbal language. This suggests that verbal representation requires the ability to connect visual and symbolic forms with logical narrative language, a skill that has not yet developed optimally in the majority of students.

The findings of this study show that students’ difficulties are not distributed evenly across visual, symbolic, and verbal indicators, and that these differences are closely related to how flexibly students are able to move between representations. The symbolic item was the most frequently left unanswered, followed by the verbal item, whereas most students at least attempted the visual task, even though the majority of their graphs were not mathematically accurate. This pattern suggests that students feel more comfortable working with concrete or procedurally familiar forms (such as plotting points on a graph) than with tasks that require them to construct algebraic models from contexts or to articulate mathematical reasoning in written language. In other words, the weakness is not only in doing mathematics, but especially in coordinating and translating between different forms of representation.

Interview data help explain why the non-response rate was so high, particularly in the symbolic and verbal indicators. Several students explicitly stated that they did not know how to begin when the problem was presented as a story or when the wording did not match the formulas they had memorized. One student, for example, admitted that although the question asked for the time when the Angry Bird hits the ground and the initial and maximum height, the student didn’t answer because the student didn’t know what formula to use. This indicates that the students did not recognize that “hitting the ground” corresponds to finding the zero of the function, or that “maximum height” is related to the vertex of the parabola, revealing a gap between contextual understanding and symbolic manipulation. Other students

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emphasized that they understood better when there's a formula and became confused when it's already in graph form, showing that they tend to treat each representation as a separate topic rather than as interconnected descriptions of the same mathematical relationship.

Affective and communicative factors also contributed to the high non-response rate, one of which is the verbal indicator. When asked to justify whose statement was correct, many students simply wrote "Fika is correct" or "Candra is correct" without giving any mathematical reason, or they copied parts of the problem statement. During the interviews, one student stated, "I know the answer, but I get confused when I have to write it down. I'm afraid of giving the wrong reason." This remark suggests that students experience not only cognitive difficulty but also low confidence in expressing mathematical ideas in written form. Their reluctance to write explanations, even when they had an intuition about the answer, indicates that the verbal item was perceived as particularly high-risk: writing an incorrect explanation felt worse than leaving the answer blank. In this sense, the non-response is not merely a sign of "not knowing," but also of uncertainty about how to translate internal understanding into acceptable mathematical language.

The interviews also revealed specific misconceptions that further hindered students' ability to connect representations. A recurring misunderstanding was the tendency to equate the maximum point (x_{max}, y_{max}) with the maximum value of the function, i.e., only the y-coordinate. One student explicitly said, "I think maximum value means the same as maximum point," and treated the ordered pair (a, b) as the maximum value. This confusion shows that students did not distinguish between the geometric object (a point on the graph) and the functional output (a single number). Similar misconceptions appeared when students tried to interpret the graph of a quadratic function. Many students drew shapes that did not resemble a parabola or connected points with straight segments because they relied mechanically on point-substitution strategies ("I just took a few x values… then I just plotted the points") without using knowledge about symmetry, axis of symmetry, or vertex. These patterns suggest that students' conceptualization of a quadratic function as a function, rather than as a set of disconnected calculation procedures, has not yet been fully formed.

When viewed through the lens of representational flexibility, these findings indicate that students' flexibility is very low. Representational flexibility involves not only being able to use different forms of representation, but also being able to move back and forth among them in meaningful ways. For example, moving from a word problem to a symbolic model, from a symbolic form to a graph, and then explaining the relationships verbally. In this study, most students appeared to operate within a single of representation. They relied heavily on numerical substitution to draw graphs but did not extend this understanding to symbolic tasks that required interpreting parameters in context, nor to verbal tasks that required explaining why two maximum points were equivalent or different. The strong overlap between failure in the symbolic and verbal indicators suggests that when students cannot construct an algebraic model, they also cannot articulate a coherent mathematical explanation, which is

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consistent with previous research reporting that students with low representational ability tend to restate problems without providing mathematically grounded justification (Utomo & Syarifah, 2021). From this perspective, the high non-response rates are a direct reflection of weak representtational flexibility: students are unable to coordinate visual, symbolic, and verbal descriptions of the same quadratic relationship.

The strategies students used further highlight the predominance of procedural rather than flexible, conceptually grounded approaches. In the visual indicator, the widespread use of point-substitution without attention to the structure of the parabola (its symmetry, vertex, and opening direction) is consistent with earlier studies that found students' errors in translating from symbolic to graphical forms stem from inappropriate construction of the target representation and poor graphing habits (Safitri et al., 2025). In the symbolic indicator, the tendency to apply factoring or the quadratic formula mechanically, without understanding the meaning of roots or the connection to the context, aligns with findings that low symbolic representation ability is often linked to difficulties in forming mathematical models during problem solving (Prayitno et al., 2021; Wardah et al., 2021; Yuhasriati et al., 2022). In the verbal indicator, students' reluctance to elaborate and their tendency to provide incomplete, opinion-based, or copied statements echo prior reports that students often write what they generally understand without explicit mathematical reasoning, particularly when their representational skills are weak, even though other studies have shown that some students can produce adequate verbal explanations when appropriately supported (Utomo & Syarifah, 2021), (Novitasari et al., 2021). Together, these patterns suggest that instruction that emphasizes technical procedures in isolation may inadvertently limit students' opportunities to develop representational flexibility.

These interpretations carry several implications for classroom practice and curriculum design. First, teachers need to deliberately design learning experiences that require students to move between visual, symbolic, and verbal modes, rather than treating each form of representation as a separate concept. For example, tasks that ask students to read a graph, express the situation verbally, and then formulate and analyze a symbolic model could help them experience representations as interconnected. Second, given students' fear of being wrong in verbal explanations, teachers should provide scaffolding for mathematical communication, such as sentence starters, structured prompts, and classroom norms that value "trying to explain" over "staying silent." Such supports may gradually build students' confidence in expressing their thinking, reducing the tendency to leave items blank. Third, it is important to address core misconceptions explicitly, such as the distinction between maximum point and maximum value, or between an ordered pair and a function output, and to connect these clarifications consistently across representations so that students see, for instance, how the vertex of a graph, the parameters in the equation, and the wording of a contextual problem all encode the same maximum.

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Finally, situating these findings within the broader literature on mathematical representation suggests that improving students' performance on quadratic function tasks requires more than adding practice items. It calls for instructional approaches that foreground the coordination of representations as a central learning goal. Prior studies have emphasized that representation is a key process standard in mathematics education and that the ability to create, use, and transform representations is fundamental to conceptual understanding and problem solving (Mainali, 2021), (NCTM, 2000), (Goldin, 2002), (Hariyani et al., 2023), (Shinariko et al., 2021), (Rahmah et al., 2022), (Fitrianna et al., 2018). The present study contributes to this line of work by illustrating how a lack of representational flexibility manifests in concrete classroom tasks. Students can sometimes carry out familiar procedures in one representation, but they struggle to connect that work to other forms or to explain it verbally. Designing learning environments that address these specific breakdowns is essential. One promising approach is to systematically link context, graph, symbol, and verbal explanation in quadratic function lessons. Such instructional design may strengthen students' representational flexibility and, ultimately, improve their understanding of quadratic functions more broadly.

Taken together, these findings suggest that students' difficulties in solving quadratic function problems are not solely caused by computational weaknesses, but by limited ability to coordinate different forms of representation. Many students were able to perform isolated procedures, such as substituting values or applying formulas, yet they struggled to interpret the meaning of these procedures across graphical, symbolic, and verbal contexts. This indicates that students often treat each representation as a separate task rather than as interconnected descriptions of the same mathematical relationship. Consequently, when the problem requires translation between representations, students experience confusion, leading either to procedural errors or to non-response. However, these findings should be interpreted with caution because the study focused only on quadratic functions and involved students from two schools, which may limit the generalizability of the results.

CONCLUSIONS

This study aimed to identify students' strategies and errors in solving mathematical representation problems on quadratic functions across visual, symbolic, and verbal indicators. The findings show that students' representation abilities are still limited. Most students attempted the visual task but produced incorrect graphs due to misconceptions about the structure of a parabola and the role of the vertex and axis of symmetry. In contrast, the symbolic and verbal indicators showed very high non-response rates, indicating that many students were unable to translate contextual information into algebraic models or express mathematical reasoning in written form. Overall, students tended to rely on procedural strategies, such as substituting values to construct graphs and recalling formulas without fully interpreting the problem context, which resulted in various errors across representations.

These findings highlight the importance of instructional approaches

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that explicitly support the coordination of multiple representations in learning quadratic functions. However, this study is limited to two schools and one mathematical topic. Future research may involve larger samples and explore additional factors that influence students' representational abilities.

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