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DIDACTICAL DESIGN RESEARCH AS A STRATEGY FOR DEVELOPING PEDAGOGICAL CONTENT KNOWLEDGE (PCK) IN MATHEMATICS TEACHERS

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Abstract

The low level of mathematics teachers' Pedagogical Content Knowledge (PCK) has become a critical issue contributing to students' learning obstacles in the topics of exponents and radical forms. This condition highlights the need for a professional development model that enhances teachers capacity to design more diagnostic and responsive instruction. This study aims to develop teachers PCK through the implementation of the Didactical Design Research (DDR) approach. A qualitative research method was employed involving eighth-grade students at SMP Negeri 1 Cigugur through DDR cycles consisting of hypothetical didactical design, implementation, retrospective analysis, and reflection. The findings indicate that the reflective DDR cycle successfully transformed teachers' instructional practices from procedural-oriented approaches to more adaptive and responsive teaching based on students learning obstacles. The revised and contextually validated empirical didactical design proved effective in addressing students fundamental conceptual difficulties. It can be concluded that DDR serves as an effective professional development model in empowering teachers as instructional designers grounded in learning obstacle analysis, while also providing a significant contribution to improving the quality of mathematics instruction at the junior secondary school level.

Keywords: Didactical Design; Didactical Design Research (DDR); Learning Difficulties; Pedagogical Content Knowledge (PCK); Professional Development

Abstrak

Rendahnya Pedagogical Content Knowledge (PCK) guru matematika menjadi permasalahan mendesak yang berdampak pada munculnya learning obstacle siswa pada materi bilangan berpangkat dan bentuk akar. Kondisi ini menunjukkan perlunya model pengembangan profesional yang mampu meningkatkan kapasitas guru dalam merancang pembelajaran yang lebih diagnostik dan responsif. Penelitian ini bertujuan untuk mengembangkan PCK guru melalui implementasi pendekatan Didactical Design Research (DDR). Penelitian menggunakan metode kualitatif yang dilaksanakan pada siswa kelas VIII SMP Negeri 1 Cigugur melalui siklus DDR yang meliputi perancangan desain didaktis hipotetik, implementasi, analisis retrospektif, dan refleksi. Hasil penelitian menunjukkan bahwa penerapan siklus reflektif DDR mampu mentransformasi praktik pembelajaran guru dari pendekatan prosedural menuju pembelajaran yang lebih adaptif terhadap hambatan belajar siswa. Desain didaktis empirik yang dihasilkan, setelah melalui proses revisi dan validasi kontekstual, terbukti efektif dalam mengatasi hambatan belajar konseptual siswa. Disimpulkan bahwa DDR merupakan model pengembangan profesional yang efektif dalam memberdayakan guru sebagai perancang pembelajaran berbasis analisis hambatan belajar, sekaligus memberikan kontribusi signifikan terhadap penguatan kualitas pembelajaran matematika di tingkat SMP.

Kata kunci: *Desain Didaktis; Didactical Design Research (DDR); Kesulitan Belajar; Pedagogical Content Knowledge (PCK); Pengembangan Profesional*

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INTRODUCTION

Building a strong foundation for students' mathematical thinking is an essential goal of learning mathematics. One of the materials considered essential in this foundation is the concept of power numbers and root forms delivered in junior high school. (Firmasari et al., 2023; Ghifari et al., 2024; Putri et al., 2024). Mastery of this material is not only the basis for understanding advanced concepts such as quadratic equations, exponent functions, and logarithms, but also plays an important role in conceptual building in algebra. Students' success in mastering this material often determines their ability to follow the next level of mathematics learning. However, challenges to achieving this success are often found in the implementation of learning, one of which is the learning approach that is not yet appropriate.

Various studies in mathematics education over the past ten years have consistently shown that teacher-centered learning approaches that emphasize procedural memorization are less effective in building students' conceptual understanding (Alabdulaziz, 2022; Hussein, 2022; Hussein & Csíkos, 2023; Koskinen & Pitkäniemi, 2022; Qetrani et al., 2021). Many teachers focus more on achieving curriculum targets than on students understanding of concepts. Thus, teachers need to master various approaches that lead students to meaningful learning (Panjaitan & Rosjanuardi, 2025).

Meaningful learning requires teachers not only to master the subject matter (content knowledge), but also to understand how students think and to anticipate their learning difficulties (Kiuhara et al., 2023; Mastuti & Prayitno, 2023). This integrated understanding enables teachers to design instruction that is responsive to students needs. This concept is known as Pedagogical Content Knowledge (PCK), a framework that underpins many studies on teacher competence.

Recent research confirms that teachers with developed PCK are able to design more effective and adaptive learning experiences, which in turn significantly improve student understanding (Apsari et al., 2020; Suharta & Parwati, 2020). In line with this, teaching methods that are responsive to students' characteristics can increase their engagement in learning, thereby supporting better achievement of educational goals. Thus, students are expected to be able to overcome their learning difficulties.

Student learning difficulties cause the achievement of educational goals to be hampered, which indicates the need for more effective strategies to overcome this problem. (Astriani et al., 2022; Johar et al., 2023; Ling & Mahmud, 2023). With a deep understanding of student characteristics, teachers can design more inclusive and responsive approaches. Thus, good PCK development is key to creating a more inclusive and effective learning environment for all students.

Although many studies have focused on analyzing student learning difficulties and teacher professional development in general, there has not been much research that systematically integrates the two in a continuous cycle. The novelty of this research lies in using the Didactical Design Research (DDR) approach as a specific framework to develop mathematics teachers' PCK directly in the classroom.

This research not only identifies problems, but also actively involves teachers in the process of designing, implementing, and reflecting on didactical

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designs to overcome real and identified student learning difficulties in the subject of power numbers and root forms.

Based on a preliminary study conducted in class VIII of SMP Negeri 1 Cigugur, a series of fundamental problems in learning practices were found. The teaching-learning process is still dominated by the expository approach, where classroom interaction tends to be one-way. As a result, students experience various conceptual difficulties (learning obstacles), such as misconceptions about powers of zero and negative powers. (for example, considers $a^0 = 1$), inability to connect the symbolic representation with its visual meaning, and procedural errors in simplifying the root form.

It was also found that teachers tend to focus primarily on procedural mastery of the content. However, they often lack insight into how students think and construct their understanding. As a result, teachers experience difficulty in anticipating and addressing the misconceptions that arise during the learning process.

To address these complex issues, this study applied the Didactical Design Research (DDR) approach. This approach is a research methodology that allows teachers to systematically analyze students' learning difficulties, design evidence-based alternative didactical designs, and conduct design trials and revisions based on real responses that occur in the classroom.

Through the DDR cycle consisting of preliminary study, didactical analysis, implementation, retrospective analysis, and reflection, teachers are facilitated to strengthen various components of their PCK in an integrated manner (Jatisunda et al., 2021; Jatmiko et al., 2021; Rahayu et al., 2021; Sidik et al., 2023).

Thus, this study aims to develop Pedagogical Content Knowledge (PCK) of junior high school mathematics teachers through the implementation of Didactical Design Research (DDR) in learning the material of power numbers and root forms. More specifically, this research is expected to produce an empirically tested didactical design to overcome students' learning obstacles, as well as formulate a model for developing teachers' professional competence that is sustainable and based on reflective classroom practice.

METHODS

This research uses a qualitative approach with the Didactical Design Research (DDR) method. This method was chosen because it aims to develop teachers' Pedagogical Content Knowledge (PCK) as well as produce effective learning designs to overcome students' learning obstacles on the material of power numbers and root forms. This research was conducted in Class VIII of SMP Negeri 1 Cigugur, Kuningan Regency. The participants consisted of 32 students who completed the written test, of whom 6 students were purposively selected to participate in in-depth interviews. The selection of the 6 students was based on their test results, representing varied levels of understanding to obtain comprehensive data on students learning obstacles.

The stages of the DDR approach consist of five main stages, namely: (1) preliminary study, (2) didactical situation analysis, (3) implementation and experimentation, (4) retrospective analysis, and (5) reflection and design. Each stage has specific objectives and procedures that are inter-related in a research cycle. Each stage is a methodological frame-work that supports the success of the learning process in the classroom (Heriyana et al., 2025).

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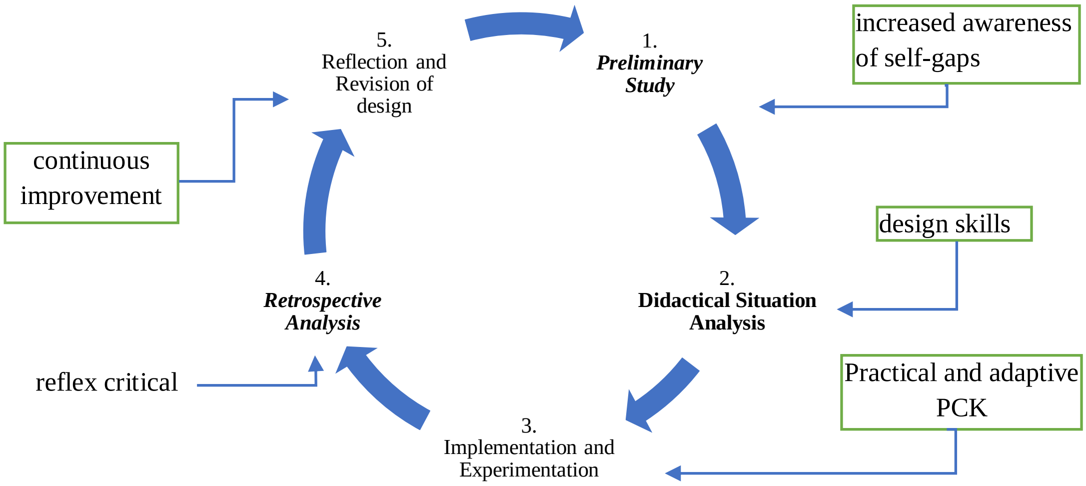

Figure 1. Integrated DDR cycle of PCK development

In Stage 1 this diagnoses the teacher's initial PCK and awareness of student learning needs and designs solutions. In stage 2 the teacher designs the HLT, actively shaping and formulating her theoretical PCK. This involves mapping students' learning obstacles and selecting teaching strategies based on a deep understanding of the content and students.

Stage 3 develops teachers' PCK in a practical, adaptive and responsive manner, as teachers learn to adjust strategies and provide conceptual explanations in real-time in front of students. In stage 4 teachers reflect and deepen their PCK by analyzing the gap between plan and reality. This changes the teacher's perspective, seeing students' difficulties as feedback to improve his or her own didactic design.

Stage 5 refines teachers' PCK by integrating new learning into planning. It also encourages a cycle of continuous improvement as better PCK becomes the starting point for the next cycle.

The instruments used in this study included a diagnostic test, semi-structured interviews, classroom observations, and document analysis, designed to identify students learning obstacles and examine teachers Pedagogical Content Knowledge (PCK) development within the Didactical Design Research (DDR) cycle. The diagnostic test was administered in written form to 32 students to assess understanding of exponents and radical forms, with indicators covering zero and negative exponents, application of exponent rules, simplification of radical forms, conceptual–procedural connections, and reasoning ability.

Interviews with selected students explored misconceptions and reasoning processes, while teacher interviews examined anticipation of learning obstacles, instructional strategies, representations, and assessment practices as components of PCK. Observations during the implementation of the Hypothetical Learning Trajectory (HLT) documented instructional practices and adaptive responses, and document analysis traced revisions of the didactical design. All instruments were validated through expert judgment to ensure content relevance, clarity, and alignment with DDR and PCK frameworks prior to implementation.

Data were analyzed qualitatively and iteratively following the stages of

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Didactical Design Research (DDR), with specific analysis techniques applied to each instrument. Students diagnostic test responses were analyzed through content analysis to identify error patterns and categorize learning obstacles into epistemological, didactical, and ontogenical types, while students interview transcripts were examined using thematic coding to clarify reasoning processes and sources of misconceptions. Observation data were analyzed descriptively to examine the alignment between the Hypothetical Learning Trajectory (HLT) and actual classroom implementation, focusing on instructional strategies, student responses, and adaptive teaching practices. Teacher interview data and document revisions were analyzed comparatively across DDR cycles to trace changes in anticipation of misconceptions, use of representations, instructional decisions, and assessment practices as indicators of PCK development.

The success indicator of the study was determined by (1) the reduction in identified learning obstacles after design revision and (2) observable improvements in teachers PCK components across cycles. Credibility of findings was ensured through triangulation of test results, interviews, observations, and document analysis.

RESULTS AND DISCUSSION

1. Preliminary Study

At the initial stage, the research identified fundamental problems in mathematics learning in class VIII of SMP Negeri 1 Cigugur. The conventional and teacher-centered teaching method was the main highlight. The problems identified were:

- **Students' Conceptual Difficulties:** Students only memorize the formulas of power numbers and root forms without understanding the basic concepts**.**
- **Representation Limitations:** Students have difficulty connecting mathematics symbols with visual or real-world contexts.
- **Lack of Real Context:** Learning is abstract and rarely linked to everyday applications.
- **Teacher PCK Limitations:** Teachers are weak in pedagogical knowledge (PK), less able to anticipate student difficulties, and less varied in teaching strategies.

Tabel 1: Summary of teacher PCK initial profile

Components PCK	Key Weaknesses
Content & Teaching Knowledge	Explanations are too theoretical, lacking concrete application examples.
Knowledge of Students	Difficulties faced by students have not been fully identified.
Learning Strategy	Limited to lectures, less active student engagement.
Learning Assessment	The questions tend to measure formula memorization, not concept understanding.

This stage fundamentally confirmed the relevance of the research title. The results of the preliminary study prove that students' learning difficulties stem from the limitations of teachers' PCK. The success of the learning process is influenced by the teacher's mastery of PCK (Suharta & Parwati, 2020). Knowledge mastery has broad dimensions, one of which is presenting meaningful information to students (Koskinen & Pitkäniemi, 2022).

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By mapping the specific weaknesses in the PCK components (as shown in Table 1), this study established a clear baseline and validated the selection of DDR as an appropriate intervention strategy, as DDR was designed to directly address and develop the identified weak aspects of PCK.

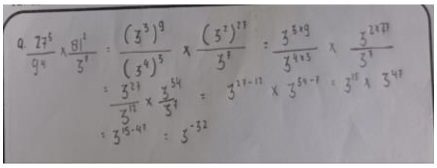
2. Didactical Situation Analysis

Based on the initial findings, this stage focuses on designing solutions by mapping out three types of learning obstacles in detail. The detail of three types of learning obstacles in detail can be seen Table 2.

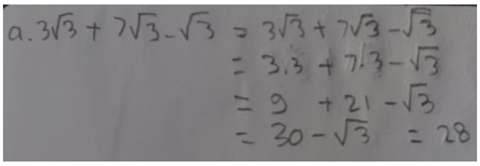
Tabel 2: Detail *Learning Obstacle*

Kategori Obstacle	Example on Material	Implications for Teacher PCK
Epistemologis	Students think $2^0=0$ (abstract concept difficulties).	Teachers need to strengthen the KCS to anticipate misconceptions.
Didactic	Difficult to visualize $8=2^2$ without media.	Teachers should develop KIS with concrete representations.
Ontogenik	Students find it difficult to relate powers to exponential growth.	Teachers need to adjust PK to the students' cognitive stage.

Students face challenges during the learning process, especially in the process of internalizing new information based on what they already know. (Dharma et al., 2021; Fauziah et al., 2023). From this analysis, the researcher developed a Hypothetical Learning Trajectory (HLT), which is a detailed learning flow for 6 meetings. The researcher carried out the implementation of learning during with the aim of identifying learning conditions, and reflecting on shortcomings, from the results of the learning that has been done. The findings in this study show that students face challenges when working on exponent problems. Student errors generally occur due to problems understanding the procedures and concepts obtained, this error is caused by various factors, one of which is the connectivity between concepts and other concepts. This factor is the basis for the emergence of algebraic misconceptions. (Johar et al., 2023).



(Errors in Exponents)



(Error in Root Form)

Figure 2. Student answer sheet

Furthermore, this stage is a form of didactical analysis. This is the first step in implementing DDR as a PCK development strategy. The process of preparing HLT forces teachers to shift from thinking about "what to teach" (content knowledge) to "how students will learn and what difficulties they will face" (pedagogical content knowledge). By mapping learning obstacles (Table 3), teachers actively practice their PCK components, especially Knowledge of

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Content and Students (KCS) and Knowledge of Instructional Strategies (KIS), before entering the classroom..

3. Implementation and Experimentation

The HLT design was piloted in the classroom in two cycles. The results showed significant progress:

- **Student Results:** There was a 28% increase in post-test scores compared to the pre-test..
- **Teacher PCK Development:** The teacher shows clear progress, moving from procedural to conceptual teaching.

Tabel 3: Teacher PCK Development Indicator

PCK aspect	Initial Implementation	End of Implementation
Student Difficulty Knowledge	Based on general experience.	Able to predict specific misconceptions.
Representasi Matematis	Using standardized examples.	Using multiple representations (visual, contextual).
Strategy Explanation	Direct explanation.	Using leading questions.
Assessment	Focus on the end result.	Pay attention to students' thought processes.

This stage demonstrates the effectiveness of DDR in developing PCK practically. The data in Table 4 clearly documents teachers' transformation from expository to constructivist methods. This development is direct evidence that the DDR cycle successfully encouraged teachers to reflect and adjust their teaching strategies in real-time. The 28% increase in student learning outcomes does not merely indicate improvement, but analytically demonstrates how the enhancement of teachers' PCK contributes to changes in instructional quality. The data suggest that as teachers become more capable of diagnosing learning obstacles and designing responsive didactical interventions, students conceptual understanding improves more substantially. Thus, teachers PCK functions as a foundational mechanism that shapes meaningful learning processes, as it directly influences instructional decisions, the anticipation of misconceptions, and the construction of students mathematical understanding (Heriyana et al., 2025).

4. Actual Didactic Situation Analysis (*Retrospective Analysis*)

A comparison between the hypothetical design (HLT) and classroom reality revealed both congruence and discrepancy: Some predictions were accurate, but students' difficulties with root forms were deeper than expected.

Tabel 4: Comparison of Prediction and Reality

Design Aspects	Initial Prediction	Reality in the Classroom	Suitability Level
Student response (power of zero)	Will have trouble understanding.	Easy to accept after seeing the pattern.	High
Difficulty (root form)	Will factorization difficulty.	Very difficult, even with guidance.	Low
Student participation	Will be passive.	Quite active in asking questions and arguing.	Currently

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Teacher Paradigm Shift: Teachers begin to see student difficulties as indicative of "learning design weak-nesses" that need to be corrected. Retrospective analysis constitutes the central mechanism of DDR as a reflective strategy for developing PCK, because it systematically compares the predicted learning trajectory with the actual classroom reality (see Table 4). This comparison does not merely prompt reflection, but analytically reveals the gaps between teachers didactical anticipations and students actual responses, thereby exposing specific weaknesses in the initial design. When teachers recognize that students difficulties originate from limitations in their didactic design rather than from students abilities alone, a critical professional awareness emerges. This shift represents a substantive deepening of PCK, as teachers move from attributing learning problems to external factors toward critically examining and reconstructing their own instructional decisions within the learning process.

5. Design Reflection and Revision

This final stage focuses on refining the design based on the findings of the analysis.

- **Concrete Revision:** The design was revised to "Empirical Didactical Design" with specific changes.
- **Teacher Competency Improve-ment and Research Legacy:** This process fundamentally improves teachers' PCK and establishes a culture of reflective and continuous improvement in schools.

Tabel 5: Learning Design Revision

Design Components	Preliminary Design	Revision
Introduction Root Forms	Directly introduces the concept.	Added a pre-learning activity on prime factorization.
Simplification Strategy	Provide the steps of the algorithm.	Added square visualization to understand the concept.
Learning Evaluation	Focus on the end result.	Adding rubrics that pay attention to the thinking process.

This stage confirms DDR as a continuous PCK development strategy. The act of revising the learning design based on empirical data (Table 5) shows a mature level of PCK. Teachers are no longer positioned merely as curriculum implementers; rather, the iterative DDR cycle situates them as evidence based learning designers whose instructional decisions are grounded in systematic analysis of classroom data. This transformation is reflected in their ability to revise didactical designs based on identified learning obstacles and empirical classroom findings. Consequently, PCK development is not treated as a static competency to be achieved, but as a continuous and dynamic professional process shaped by cycles of planning, implementation, analysis, and redesign.

The findings indicate that the implementation of Didactical Design Research (DDR) is closely aligned with the theoretical framework of Pedagogical Content Knowledge (PCK), particularly in integrating content mastery with pedagogical responsiveness to students learning obstacles. The 28% increase in student learning outcomes supports the initial assumption that strengthening teachers

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PCK through reflective and iterative cycles positively impacts students conceptual understanding. Thus, the results are consistent with the study's conceptual foundation and research objective. This gap analysis led to revisions in instructional strategies, task structures, and conceptual representations. As teachers became more aware of the sources of students misconceptions, their instructional decisions became more adaptive and diagnostic, resulting in improved learning outcomes.

The key findings of this study include: (1) retrospective analysis as a central mechanism for deepening teachers professional awareness; (2) the development of an empirically revised didactical design effective in addressing conceptual obstacles in exponents and radical forms; and (3) the transformation of teachers roles from procedural instructors to evidence-based learning designers.

These outcomes were influenced by structured DDR cycles, the use of diagnostic data (tests and interviews), and sustained reflective practice. This study's strength lies in its integration of PCK theory with a systematic professional development model and empirical classroom evidence. However, its limitation includes the restricted research context (one class in a single school) and focus on a specific mathematical topic, which may limit broader generalization.

The implications of this study suggest that PCK should be viewed as a dynamic and continuously developed competence. Practically, DDR can serve as an effective model for teacher professional development, particularly in designing instruction grounded in systematic analysis of students learning obstacles.

CONCLUSIONS

This study concludes that the implementation of Didactical Design Research (DDR) effectively develops teachers Pedagogical Content Knowledge (PCK). Through its systematic and iterative cycle, DDR enhances teachers ability to anticipate students learning obstacles, select appropriate instructional strategies, and revise didactical designs based on classroom evidence. The key finding of this study is that the reflective DDR process transforms teachers into evidence based learning designers rather than mere content deliverers. Therefore, DDR contributes as a dynamic professional development model that supports the continuous and structured growth of teachers PCK.

Although this study showed significant results, there are some limitations that need to be acknowledged. The scope of the research, which is a case study on one teacher, in one school, and on one specific material topic, limits the generalizability of these findings to a broader context. The success of teachers' PCK development is also strongly influenced by intensive assistance from researchers, so the effectiveness of this model under normal conditions without external intervention is still in question. In addition, this study has not measured the sustainability of PCK improvement and changes in teachers' teaching practices in the long term after the formal intervention program ends, so further studies are needed to ensure a permanent impact.

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REFERENCES

Alabdulaziz, M. (2022). The effect of using PDEODE teaching strategy supported by the e-learning environment in teaching mathematics for developing the conceptual understanding and problem-solving skills among primary stage students. *Eurasia Journal of Mathematics, Science and Technology Education*, *18*(5). https://doi.org/10.29333/ejmste/12019

Apsari, R. A., Sariyasa, S., Indrawan, G., Maulyda, M., & Radiusman, R. (2020). Why should you reverse the order when dividing a fraction? A study of pre-service mathematics teachers' pedagogical content knowledge in fractional concept. *Journal of Physics: Conference Series*, *1503*. https://doi.org/10.1088/1742-6596/1503/1/012019

Astriani, L., Mujib, A., & Firmansyah. (2022). Didactic Design for Overcoming Learning Obstacles in Mathematics of Junior High School Students. *International Journal of Educational Research Excellence (IJERE)*. https://api.semanticscholar.org/CorpusId:251429710

Dharma, D., Kamid, K., & Yantoro, Y. (2021). Analyzing Learning Obstacle with Didactical Design Research on Three Dimensional Distance Material. *Indonesian Journal of Science and Mathematics Education*. https://api.semanticscholar.org/CorpusId:245778118

Fauziah, E., Lidinillah, D. A. M., & Apriani, I. (2023). Obstacle to Learning Algebra in Elementary Schools. *AlphaMath : Journal of Mathematics Education*. https://api.semanticscholar.org/CorpusId:265598987

Firmasari, S., Herman, T., & Nurlaelah, E. (2023). The evolution of Indonesian curriculum: Hypothetical learning trajectory for mastery of mathematical and computational thinking. *JRAMathEdu (Journal of Research and Advances in Mathematics Education)*, *8*(4), 213–224. https://doi.org/10.23917/jramathedu.v8i4.2116

Ghifari, M. T., Nurjanah, N., Herman, T., & Rahmah, H. (2024). Challenges In Developing Junior High School Students' Computational Thinking Skills In Mathematics Through Problem-Based Learning. *Kalamatika: Jurnal Pendidikan Matematika*, *9*(2), 135–146. https://doi.org/10.22236/KALAMATIKA.vol9no2.2024pp135-146

Heriyana, T., Umbara, U., & Puadi, E. F. W. (2025). Didactical design research on mathematical sequence material in vocational high schools for job preparation. *Jurnal Elemen*, *11*(2), 277–296. https://doi.org/10.29408/jel.v11i2.27753

Hussein, Y. (2022). Conceptual Knowledge and Its Importance in

DOI: https://doi.org/10.24127/ajpm.v15i1.14342

Teaching Mathematics. *Middle Eastern Journal of Research in Education and Social Sciences*, *3*(1). https://doi.org/10.47631/mejress.v2i4.327

Hussein, Y., & Csíkos, C. (2023). The effect of teaching conceptual knowledge on students' achievement, anxiety about, and attitude toward mathematics. *Eurasia Journal of Mathematics, Science and Technology Education*, *19*(2). https://doi.org/10.29333/ejmste/12938

Jatisunda, M. G., Suryadi, D., & Prabawanto, S. (2021). Development of competencies for teaching geometry through an analysis learning obstacle. *Journal of Physics: Conference Series*, *1806*. https://doi.org/10.1088/1742-6596/1806/1/012114

Jatmiko, M. A., Herman, T., & Dahlan, J. A. (2021). Desain Didaktis Materi Kaidah Pencacahan Untuk Siswa SMA Kelas XI. *Hipotenusa Journal of Research Mathematics Education (HJRME)*. https://doi.org/10.36269/hjrme.v4i1.464

Johar, R., Desy, D., Ramli, M., Sasalia, P., & O, H.-C. (2023). Preservice Teachers' Noticing Skills in Relation to Student Misconceptions in Algebra. *European Journal of Educational Research*, *12*(2), 865–879. https://doi.org/10.12973/eu-jer.12.2.865

Kiuhara, S. A., Levin, J., Tolbert, M., O'Keeffe, B. V., O'neill, R., & Jameson, J. (2023). Teaching argument writing in math class: Challenges and solutions to improve the performance of 4th and 5th graders with disabilities. *Reading and Writing*, 1–30. https://doi.org/10.1007/s11145-023-10459-7

Koskinen, R., & Pitkäniemi, H. (2022). Meaningful Learning in Mathematics: A Research Synthesis of Teaching Approaches. *International Electronic Journal of Mathematics Education*, *17*(2). https://doi.org/10.29333/iejme/11715

Ling, P. M., & Mahmud, M. S. (2023). Pedagogical Content Knowledge on Primary School Mathematics Teachers Based on Specialization. *International Journal of Academic Research in Progressive Education and Development*, *12*(2), 1033–1048. https://doi.org/10.6007/IJARPED/v12-i2/16960

Mastuti, A. G., & Prayitno, L. L. (2023). Exploring high school teacher's design of rich algebra tasks. *Jurnal Elemen*, *9*(1). https://doi.org/https:/doi.org/10.29408/jel.v9i1.5851

Panjaitan, M. A., & Rosjanuardi, R. (2025). Identifikasi hambatan belajar siswa pada materi konsep persamaan dan pertidaksamaan nilai mutlak satu variabel. *Aksioma: Jurnal Program Studi Pendidikan Matematika*, *14*(3), 702. Https://doi.org/10.24127/ajpm.v14i3.7180

Putri, C. K., Prabawanto, S., & Juandi, D. (2024). Students' Errors in Solving Powers and Roots Based on AVAEM (ARITH, VAR, AE, EQS, and MATH) Categories. *KnE Social Sciences*, 1355–1363.

DOI: https://doi.org/10.24127/ajpm.v15i1.14342

https://doi.org/10.18502/kss.v9i13.16075

Qetrani, S., Ouailal, S., & Achtaich, N. (2021). Enhancing Students' Conceptual and Procedural Knowledge Using a New Teaching Approach of Linear Equations Based on the Equivalence Concept. *Eurasia Journal of Mathematics, Science and Technology Education*. https://doi.org/10.29333/ejmste/10938

Rahayu, E. S., Juandi, D., & Jupri, A. (2021). Didactical design for distance concept in solid geometry to develop mathematical representation ability in vocational high school. *Journal of Physics: Conference Series, 1882*. https://doi.org/10.1088/1742-6596/1882/1/012077

Sidik, G. S., Zahrah, R. F., Hariyani, M., & Fadhilaturrahmi, F. (2023). Development of Mathematics Teaching Materials Using Didactical Design Research: A Study on Enhancing Pedagogical Content Knowledge in Quadratic Inequalities. *Journal of Education For Sustainable Innovation, 1*(1). https://doi.org/10.56916/jesi.v1i1.489%2039

Suharta, I., & Parwati, N. N. (2020). Relationship Between Teacher's Content Knowledge, Pedagogical Content Knowledge, and Self-Efficacy and Its Impact on Student's Mathematics Learning Achievement. Proceedings of the 4th Asian Education Symposium (AES 2019). https://doi.org/10.2991/assehr.k.200513.066