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EFFECTIVENESS OF RMT-BASED LEARNING TRAJECTORIES TO SUPPORT RELATIONAL UNDERSTANDING

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Abstract

Straight line equations are one of the mathematical materials that are often difficult to understand. This difficulty is due to a teaching approach that often emphasizes memorizing formulas without a deep understanding, as well as a lack of reasoning and problem-solving exercises. This study aims to design a learning trajectory of straight line equations using the Rigorous Mathematical Thinking (RMT) approach and its effectiveness on students' relational understanding. The research method used is design research, which focuses on designing a learning trajectory and consists of three stages: preliminary analysis, experiment, and retrospective which are carried out in two cycles. The subjects of this study were 8th grade students of Palembang State Junior High School, with 32 students as participants in the pilot experiment and 36 students as participants in the teaching experiment, which were selected by purposive sampling. Data is collected through observation, written tests, and interviews. The data from the HLT design will be analyzed qualitatively and *the results of the pre-test* and *post-test* will be analyzed quantitatively. The results of this study produce a learning trajectory consisting of two main activities i.e Solving problems related to line equations that go through two points, and perpendicular line equations. The learning trajectory developed is able to support students' relational understanding, where in the learning process, students are directed to utilize the various knowledge they have, determine the most effective and efficient strategies, provide detailed solutions, and believe in the truth of the answers they use in solving problems. In addition, pre and post test results, there is an increase in relational understanding, it can be concluded that the RMT-based learning trajectory is effective for improving students' relational understanding of students in learning straight line equation material.

Keywords: Learning design, relational understanding, rigorous mathematical thinking, straight line equations.

Abstrak

Persamaan garis lurus merupakan salah satu materi matematika yang sering kali sulit dipahami. Kesulitan ini disebabkan oleh pendekatan pengajaran yang sering kali menekankan pada hafalan rumus tanpa pemahaman yang mendalam, serta kurangnya latihan penalaran dan pemecahan masalah. Penelitian ini bertujuan untuk merancang lintasan pembelajaran persamaan garis lurus menggunakan pendekatan Rigorous Mathematical Thinking (RMT) serta efektivitasnya terhadap pemahaman relasional siswa. Metode penelitian yang digunakan adalah penelitian desain, yang berfokus pada perancangan lintasan pembelajaran dan terdiri dari tiga tahap: analisis pendahuluan, eksperimen, dan retrospektif yang dilakukan dalam dua siklus. Subjek penelitian ini adalah siswa kelas 8 SMP Negeri Palembang, dengan 32 siswa sebagai peserta pilot eksperimen dan 36 siswa sebagai peserta eksperimen pengajaran, yang dipilih secara purposive sampling. Data dikumpulkan melalui observasi, tes tertulis, dan wawancara. Data hasil rancangan HLT akan dianalisis secara kualitatif dan hasil pre-test dan post-test akan dianalisis secara kuantitatif. Hasil penelitian ini menghasilkan lintasan pembelajaran yang terdiri dari dua kegiatan utama yaitu Menyelesaikan masalah yang berkaitan dengan persamaan garis yang melalui dua titik dan persamaan garis yang tegak lurus. Lintasan pembelajaran yang dikembangkan ini mampu mendukung pemahaman relasional siswa, dimana dalam proses pembelajaran siswa diarahkan untuk memanfaatkan berbagai pengetahuan yang dimiliki, menentukan strategi yang paling efektif dan efisien, memberikan solusi yang detail, serta meyakini kebenaran jawaban yang mereka gunakan dalam menyelesaikan masalah. Selain itu, hasil yang didapatkan adalah hasil pre-test dan post-test terdapat peningkatan pemahaman relasional, maka dapat disimpulkan lintasan belajar berbasis RMT efektif untuk meningkatkan pemahaman relasional siswa dalam pembelajaran materi persamaan garis lurus.

Kata kunci*: Desain pembelajaran, pemahaman relasional, persamaan garis lurus, Rigorous mathematical thinking.*

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INTRODUCTION

Straight line equation is an important mathematical material at the junior high school level, which plays a role in developing the concept of function that has been learned previously (Devi, 2018). Material concept straight line equation is an essential material that contains several materials in it such as algebra and measurement (Hakim & Aji, 2022). This material needs to be studied in depth to avoid conceptual errors (Tashtoush & Wardat, 2023). Therefore, the delivery of the concept of the equation of a straight line must be done coherently, from gradient to determining the equation through one point and gradient or two points. However, learning this topic is often dominated by memorization of formulas without in-depth understanding, thus not training students' reasoning and problem-solving skills (Ibrahim, 2020). For this reason, students must have the ability to understand the problems given.

Understanding is a key factor in mathematics to achieve learning goals. Mathematical understanding which plays an important role in linking one concept with another concept turns out to be still a problem for students (Hidaiyah, Sukoriyanto, & Slamet, 2023). In the context of mathematics, understanding is divided into relational and instrumental understandin (Amir, Wardana, & Usfuriah, 2021). Relational understanding refers to students' awareness of the appropriate steps and logical reasoning behind the choice of problem-solving procedures. In contrast, instrumental understanding only involves the ability to use concepts without understanding the reasoning. This research will focus on developing relational understanding.

Relational understanding is very important because it allows a person to understand the reasons behind each procedure, not just being able to use concepts (Utomo & Huda, 2020). Relational understanding in math offers four key advantages, it simplifies complex problem-solving, boosts memory and comprehension of concepts, facilitates achieving learning objectives, and fosters original thinking. Based on this, relational understanding is very necessary for students to have, including in the material of straight line equations.

Based on observations and interviews with mathematics teachers, it is found that the learning carried out so far has not directed students to understand mathematical concepts deeply. This is in line with Samosir, Dahlan, Herman, & Prabawanto (2023), regarding relational understanding, which states that learning activities still do not support students to understand concepts relationally because they are only given formulas and practice problems and assignments. In addition, also stated that students' relational understanding is still low, this is characterized by students' inability to identify the reasons why the procedures used in solving problems (Rachmawati, Subanti, & Usodo, 2023). Thus, learning is needed that can help students connect between mathematical concepts and understand why a procedure is used in solving problems.

One way that can be done is to apply learning based on the Rigorous mathematical thinking (RMT) approach, because RMT is one of the approaches that is centered on teachers and students. RMT helps students identify problems and figure out solutions (Hayati, Astuti, & Febrian, 2022). The RMT approach has been proven effective in improving high-level

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mathematical thinking skills as well as improving other skills (Firmasari et al., 2022). This approach aims to make mathematics more accessible and applicable for students in their everyday situations (Syafri, Susanta, & Koto, 2024). Based on the results of (Nuraisyah, Pratiwi, & Aisyah, 2023), the RMT approach can be used in mathematics learning because the activities of teachers and students in mathematics learning by applying the RMT approach obtained very good criteria.

Previous research related to learning design using RMT has been conducted on the topic of geometry to support qualitative thinking in RMT, Pratiwi et al's research focuses on only one of the three levels of thought in the RMT approach (Pratiwi, Hauda, Kurniadi, Araiku, & Astuti, 2022). In addition, learning tools have also been

designed using RMT but on relationship and function materials (Ayu, 2019). They suggest that other researchers develop and use the RMT approach to improve other students' abilities, such as concept comprehension, critical thinking skills, and critical thinking skills.

Researchers use the RMT approach to support relational understanding of straight-line equation material, as RMT can help students develop a deeper understanding of straight-line equations through a more formal and structured approach. By using RMT, students can learn and understand the concept of straight line equations more thoroughly and in detail.

In addition, by using a learning process with three phases and six steps, RMT can direct students in improving their relational understanding. The first phase is Cognitive Development, 1) Students adapt models or ways of thinking to solve cognitive tasks related to straight line equations, 2) Through stimuli provided by teachers, students use psychological tools such as using symbols, diagrams or graphs in carrying out cognitive tasks on straight line equation material. The second phase is Content Development as a Process, 3) Students build basic concepts related to line slope (gradient) and straight line equations based on two known points and lines perpendicular to other lines, 4) Through the guidance of the teacher, students find patterns or formulas to solve a given problem, 5) Students adapt certain mathematical psychological tools, so that they can think of other ways to solve the given problem. The third phase is Cognitive Conceptual Practice, 6) Students are asked to solve the problem independently or in groups, the teacher is only the facilitator.

Based on several studies that have been conducted related to RMT, there has been no research that designs learning using RMT to support relational understanding. Based on the background that has been described, there is an urgent need to improve the quality of mathematics learning, especially in supporting students' relational understanding. Therefore, the researcher focused on producing a learning trajectory for straight line equations using the Rigorous Mathematical Thinking (RMT) approach.

METHODS

This study employs design research, focusing on the design of learning trajectories with the goal of developing and validating theories about the learning process and how learning trajectories can be constructed (Plomp & Nieveen, 2006). Figure 1 shows the research flow chart.

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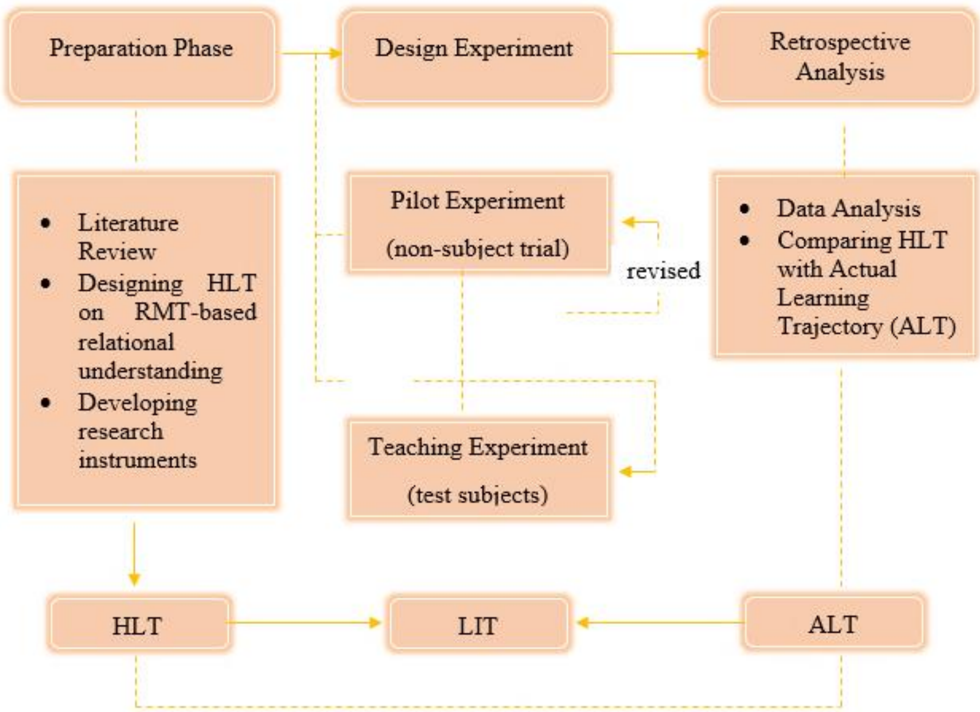

Figure 1. Research flowchart

Validation studies typically involve a preparation phase, a design experiment (comprising pilot and teaching experiments), and a retrospective analysis (Gravemeijer & Eerde, 2009). During preparation, researchers conduct literature reviews and develop both the learning trajectory (HLT) and research instruments. The pilot experiment tests and refines the HLT, assessing its effectiveness and instrument implementation. Subsequently, the teaching experiment applies the improved HLT to a target classroom.

This study engaged eighth-grade students from a Palembang junior high school. Participants were purposively sampled based on teacher recommendations and availability. Two classes participated: one of 32 students for the pilot experiment, and another of 36 for the teaching experiment. A mathematics teacher and a research assistant also contributed to the study.

The data for this study came from observations, written tests, and interviews. Classroom activities were documented through observation sheets and video recordings. Student attitudes and skills were assessed using predetermined criteria. Learner worksheets (LKPDs) provided written test data, analyzed for student conjectures, solution strategies, and learning outcomes. Narrative descriptions complemented LKPD analysis. Semi-structured interviews, adapted to student responses, were conducted to gather additional insights.

The data from the HLT design will be analysed qualitatively and the pre-test and post-test results will be analysed quantitatively. These narratives, in conjunction with observational data, provided a comprehensive understanding of student LKPD performance. Descriptive analysis was employed to characterize student thinking during the learning process and the emergence of relational understanding. Indicators of relational understanding were identified based on

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the framework of Huda & Utomo (2020) and Kilpatrick, Swafford, and Findell (2001) as presented in Table 1.

Table 1. Indicators of relational understanding

Indicators of Relational Understanding	Descriptor
Ability to perform detailed procedures	• Students can write down what they know from the problem given. • Students can write down what is the problem.
Obtaining the right result	• Students can write the correct answer • Students can write conclusions from the results they have obtained
Able to use formula that are appropriate to the problem	• Students can use appropriate and appropriate methods to solve problems
Recognising new forms of problems that can be solved using procedures	• Students can solve new problems using the same procedures according to the concept of equation of a straight lines.

RESULTS AND DISCUSSION

Preparation Phase

In the preparation phase, a comprehensive literature review was conducted to identify common difficulties students face with equation of a straight lineand to explore the Rigorous Mathematical Thinking (RMT) approach. Hypothetical Learning Trajectories (HLT) were designed, including lesson plans, learning materials, and assessment tools. These materials were subjected to expert review by two mathematics educators to evaluate their clarity, pertinence, and efficacy. Figure 2 visually represents the HLT for linear equations.

The HLT design includes learning objectives, activities and learners' thinking conjectures. The set of activities designed for learning straight line equations in the learning trajectory is described as follows.

Design Experiment

The design experiment comprised two cycles. The initial cycle, a pilot experiment involving 32 eighth-grade students, aimed to test the feasibility of the initial HLT and inform necessary refinements based on observations and feedback.

Pilot Experiment

The pilot experiment was conducted with a non-subject class of 32 eighth-graders. This phase involved three sessions. The first session focused on determining the equation of a line passing through two given points, including graphical representation and coordinate identification. The second session explored the equation of a line perpendicular to another, emphasizing gradient calculation and application of the relevant formula. The final session provided an opportunity for knowledge consolidation and addressing student questions.

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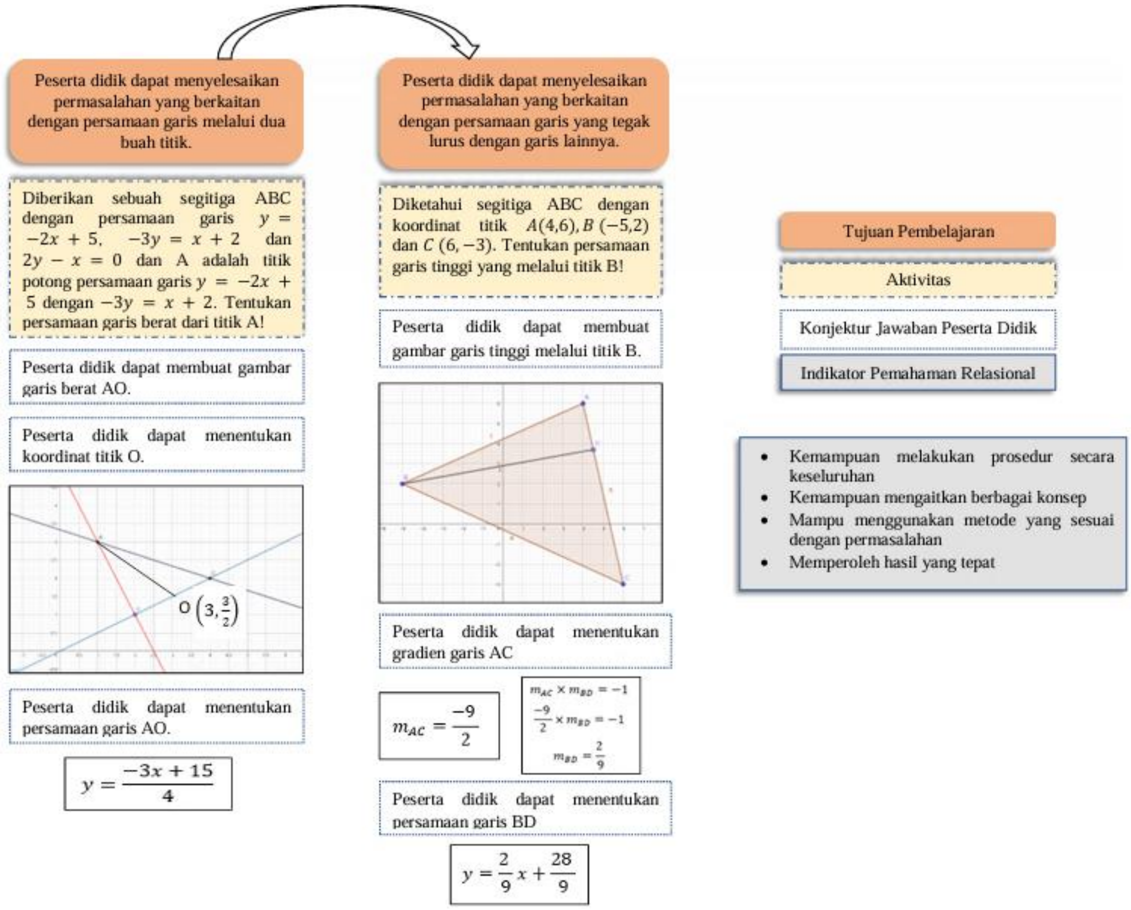

Figure 2. HLT line equation

Upon completion of Cycle 1 activities, it became evident that students encountered difficulties with algebraic manipulations and connecting the equations to their geometric representations. Feedback highlighted the need for more explicit teacher guidance on specific concepts. In response, the HLT and associated materials were refined for improved clarity and effectiveness. Figure 3 provides a visual representation of the pilot experiment implementation.

The pilot experiment findings informed a review of the HLT, particularly in light of student conjectures evidenced in the LKPDs. Based on these insights, the HLT underwent further refinement to address identified shortcomings. The revised HLT was subsequently prepared for testing in the next phase: the teaching experiment.

Figure 3. Pilot experiment stage

Pilot experiment results indicated that students disregarded instructions to redraw the weight line in Problem 1, drawing directly on the provided image instead. To address this, the teaching experiment will explicitly require students to redraw the line in a designated area. Additionally, as the

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pilot revealed no student use of the two-point line equation formula, the teaching experiment will adopt a more flexible approach, allowing for alternative solution strategies. Learners are allowed to find the equation of the line using the formula $y - y_1 = x - x_1$.

Teaching Experiment

Pre-test

Researchers investigated the initial ability of students after conducting a literature review and determining research subjects by giving a written test in the form of a pre-test. The pre-test was given with the aim of obtaining information about learners' understanding of straight line equation material before carrying out the designed learning activities. The pre-test results were used to develop a series of learning activities and adjust the HLT if deficiencies were found. The pre-test consisted of two description questions that were done individually by the students. Figure 4 shows the pre-test questions given to students.

1. Tentukan persamaan garis yang melalui titik koordinat dibawah ini
   a. $(2, -1)\ dan\ (-5,4)$
   b. $(a, 0)\ dan\ (0, b)$
2. Tentukan persamaan garis yang melaluti titik $(-1,2)$ dan tegak lurus terhadap garis $4y = -3x + 5$.

Figure 4. *Pre-test*

The pre-test questions given were two questions. The first question was about the equation of a line through two points and the second question was about the equation of a line perpendicular to another line. Table 2 shows the pre-test results.

Table 2. Pre-test result

Descriptive	Statistic	Std. Error
Mean	68,69	1,027
Median	69,00	
Variance	30,579	
Std. Deviation	5,530	
Minimum	60	
Maximum	80	
Range	20	

Based on Table 2, the pre-test results showed that the average score of students was 68.69 with the highest score of 80 and the lowest score of 60.

Activities

The refined HLT was implemented in a teaching experiment involving 36 eighth-grade students. The primary objective of this phase was to assess the revised HLT's efficacy in fostering students' relational understanding of linear equations.

Activity 1: Finding the line equation from two points.

The teacher gives LKPD Activity 1 to the 36 students, divided into nine groups. The lesson commenced with a clear articulation of learning objectives and an exploration of students' prior knowledge about linear equations. Students were then tasked with understanding the LKPD problems, which focused on determining the equation of a triangle's median. This activity capitalized on students' existing knowledge of triangle medians. Figure 5 provides a detailed overview of LKPD Activity 1 for the first meeting.

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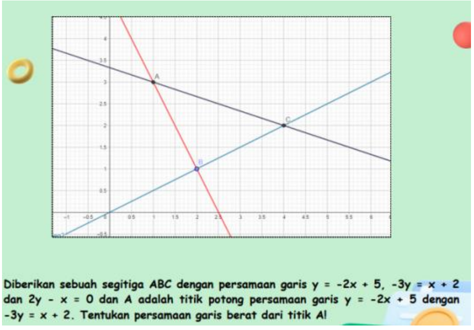

Figure 5. LKPD activities 1

Students were instructed to solve the problems sequentially as outlined in the LKPD. The initial task involved drawing a median from point A to intersect line BC at point O, followed by determining point O's coordinates. Figure 6 presents a visual representation of students' responses to this task.

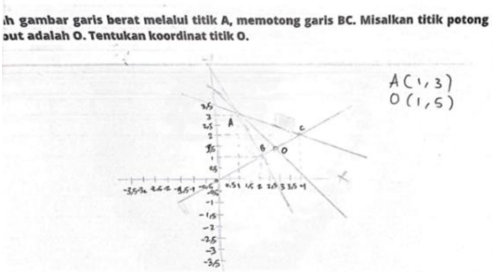

Figure 6. Results of student's answer number 1

Based on the results of the learners' answers, learners can draw a heavy line through point O and cut the BC line. Learners can also determine the coordinates of point O, namely (3;1.5) by observing the picture given in the problem and seeing the x and y coordinates corresponding to point O. Figure 7 shows the learners' answers to the second question.

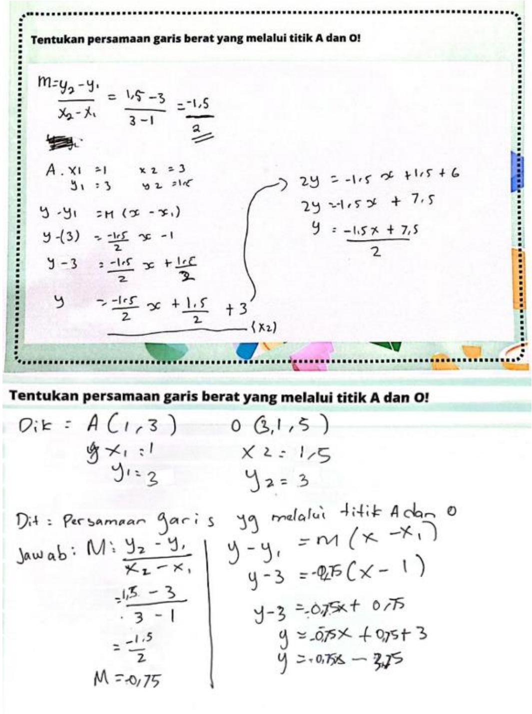

Figure 7. Results of student's answer number 2

Analysis of student responses revealed a strong ability to determine the equation of a line passing through two points, specifically points A and O. Students initially calculated the line's gradient before deriving the equation. Their choice of formula, given a point and gradient, aligned with previously taught concepts by the mathematics teacher.

The answers from all groups were consistently alike, showing no significant variation. All groups used the same formula to get the equation of the line, namely $y - y_1 = x - x_1$. However, there were differences in writing the answers. As shown in Figure 4, the first group wrote the result in fraction form while the second group wrote it in decimal form, but both answers were the same and correct.

Activity 2: Finding perpendicular lines.

To initiate Activity 2, students were informed of the learning

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objectives and their prior knowledge was activated. Figure 8 provides a detailed outline of LKPD 2 activities.

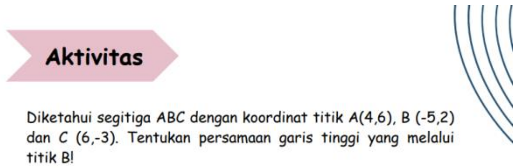

Figure 8. LKPD activities 2

In Activity 2, students were tasked with determining the equation of a line perpendicular to another. Figure 9 illustrates their responses to the initial problem.

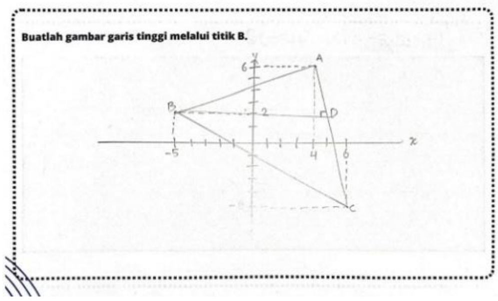

Figure 9. Results of student's answer number 1

In the first question, students were instructed to draw an altitude from point B, labeling the intersection point as D. The segment BD thus represented the altitude.

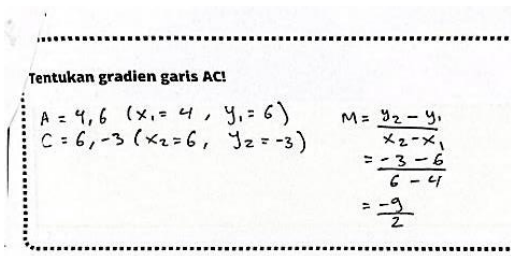

Figure 10. Results of student's answer number 2

Figure 11 shows the answer to the second question, namely learners are asked to determine the gradient of the AC line. The answer given by students is correct, namely to determine the gradient of the AC line using points A and C so that the gradient of the AC line is equal to $\frac{-9}{2}$.

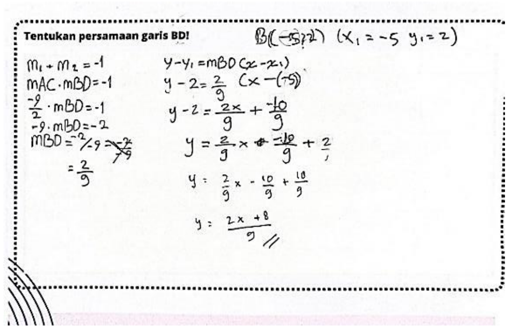

Figure 11. Results of student's answer number 3

In the third question as shown in Figure 11, learners are asked to determine the equation of the line BD. Line BD is the height line given in the problem. Because the BD line is perpendicular to the AC line, the gradient used is $mAC \times mBD = 1$, so we get $mBD = \frac{2}{9}$. After that, to determine the equation of the BD height line using the gradient of the BD line. So the line equation is $y = \frac{2x+8}{9}$. Just like in the first activity, there was no significant difference in learners' answers. Learners solve problems using the same formula, but have differences in the results obtained. As in Figure 11, the left learner's answer shows that there is an operation error in calculating the formula $\frac{2}{9}(x-(-5))$ where the answer should be $\frac{2}{9}x + \frac{10}{9}$.

Post-test

The post-test was conducted after the series of activities in the teaching experiment was completed. The post-test was given to determine the effectiveness of the activities designed to support relational understanding. The following post test questions are given in Figure 12.

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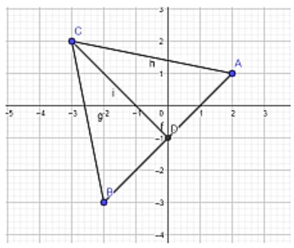
Kerjakanlah soal dibawah ini dengan benar.

1. Perhatikan gambar dibawah ini!



Tentukan persamaan garis CD!

2. Garis h memotong sumbu x positif di A dan sumbu y positif di B. Jika O adalah titik pangkal sistem koordinat (0,0), $AO = 3$ dan $OB = 4$, maka tentukan persamaan garis g melalui O dan tegak lurus pada h!

Figure 12. Post-test

The post-test featured two problems. One required finding the equation of a line passing through two given points, and the other involved determining the equation of a line perpendicular to a given line. Table 3 shows the post test results.

Table 3. Post-test Result

Descriptive	Statistic	Std. Error
Mean	78,10	2,053
Median	85,00	
Variance	122,239	
Std. Deviation	11,056	
Minimum	60	
Maximum	90	
Range	30	

After learning using the RMT approach, the results obtained are in accordance with Figure 14 that the average score of students increased from 68.69 to 78.10, with the highest score of 90 and the lowest score of 60.

Effectiveness of Learning Trajectory

This research aimed to design an RMT-based learning trajectory for linear equations. By leveraging students' prior knowledge, the RMT effectively guided learners through the problem-solving process. Results indicate that students successfully executed procedures, integrated concepts, employed suitable methods, and achieved correct solutions. Nevertheless, students effectively applied knowledge to new contexts, as evidenced by their strategic use of gradient to determine equations (Hidayat et al, 2021).While accurate solutions were obtained, students often omitted concluding statements. This study underscores the potential of RMT-based learning trajectories to enhance mathematics instruction. Teachers are encouraged to adopt similar iterative design approaches to optimize teaching practices and materials.

The results demonstrated a significant enhancement in students' relational understanding, characterized by their ability to interconnect mathematical concepts and articulate the rationale behind their problem-solving approaches. Students exhibited increased proficiency in selecting appropriate methods and explaining conceptual connections. Furthermore, Mefiana & Juandi (2023) emphasized the pivotal role of relational understanding in tackling complex mathematical problems. The findings underscore the potential of the RMT approach in cultivating deeper conceptual understanding. To maximize its impact, educators should prioritize learning environments that foster exploration, inquiry, and conceptual connections, thereby promoting more profound and enduring learning outcomes.

The hypothesis test used in this study is a t-test (paired sample t-test) with the help of SPSS. Testing this hypothesis is carried out to find out the provisional conjecture formulated by the researcher.

$Ho: \mu_1 = \mu_2$:

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There was no improvement in students' learning outcomes towards relational understanding.

$Ha: \mu_1 \neq \mu_2$

There is an increase in students' learning outcomes towards relational understanding.

μ_1: mean of *pre-test*

μ_2: mean of *post-test*

Before the t-test is carried out, the average *pre-test* and *post-test* are first sought.The results obtained are as follows as shown in Table 4.

Table 4. Mean of pre-test and post-test

	Mean	N	Std Dviation	Std. Error Mean
Pretest	68,69	29	5,530	1,027
Postest	78,10	29	11,056	2,053

The test criteria are if the value $t_{hitung} > t_{tabel}$ or Sig < 0.05 then it is Ho rejected and Ha accepted, but if or Sig > 0.05 or $t_{hitung} < t_{tabel}$ then it is accepted and rejected. The results of the Paired sample t-test are as follows in Table 5.

Table 5. Paired sample t-test

Pengujian	t_{hitung}	t_{tabel}	Sig
Uji t	5,072	2,048	0,01

Based on table 4, it shows a value of 5.072, the Sig level = 0.01. With a value of up to 2,048 at the level . Based on the results above, it can be concluded that it is rejected and accepted because, namely. So there is an increase in students' learning outcomes towards relational understanding. $df = 28 \propto = 0{,}05 Ho\ Ha t_{hitung} > t_{tabel} 5{,}072 > 2{,}048$.

The results of this study significantly showed that the application of Rigorous Mathematical Thinking (RMT) based learning succeeded in improving students' relational understanding, as evidenced by the increase in the average pre-test and post-test scores using SPSS analysis. This improvement can be explained because RMT encourages students to not only learn facts or formulas in isolation, but to identify and build logical relationships between various mathematical concepts, in line with the essence of deep mathematical thinking. The RMT learning trajectory designed in this study effectively facilitated adaptive thinking and concept connection. For example, in the post-test questions, students are faced with the challenge of applying the concept of line first before using the formula for the equation of a straight line, which demands an understanding of "why" and "how" a concept applies, not just "what" the formula is. This approach is aligned with RMT's goal of developing structured and logical mathematical thinking.

This research is in line with other researchers who say the RMT approach is very effective in learning, the thing that distinguishes it is the focus of the abilities studied. The results of Syafri et al., (2024) research explained that the RMT approach effectively integrates deep mathematical thinking with relevant cultural contexts, presenting an innovative approach to teaching mathematics. Other than that in Amalia & Hidayati (2024), RMT-based learning influence the ability of concept understanding, spatial reasoning, and self-regulated learning of students.

This increase in students' relational understanding occurs for several main reasons in RMT. RMT

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places great emphasis on deep problem solving, critical thinking skills, and the way students build strong mathematical arguments. It helps students see concepts from multiple sides and connect mathematical ideas to each other. This is in line with Firmasari & Juandi (2021) that thinking mathematical rigor is able to lead rigor, perseverance, critical inquiry, and the search for truth in solving problems precisely, structured, and systematically. Therefore, teachers should focus on creating a learning environment that encourages learners to explore, question and connect mathematical ideas. This can result in more meaningful and long-lasting learning outcomes.

CONCLUSION AND SUGGESTION

The developed learning trajectory outlines a step-by-step learning process, encompassing two primary activities: determining the equation of a line through two points and finding the equation of a perpendicular line. Results indicate that the designed and tested learning path effectively facilitated student comprehension and fostered the development of relational understanding. Through the LKPD activities, students demonstrated enhanced problem-solving abilities, including method selection, adaptation to novel problem formats, and accurate solutions.

Based on the results and discussions related to the effectiveness of relational understanding, it can be concluded that it is Ho rejected and Ha accepted because, namely, $t_{hitung} > t_{tabel} 5{,}072 > 2{,}048$. Based on the pre and post test results, there is an increase in relational understanding, it can be concluded that the RMT-based learning trajectory is effective on students' relational understanding.

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