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# STUDENTS' ERRORS IN TRANSLATING MATHEMATICAL REPRESENTATIONS FROM SYMBOLIC TO GRAPHICAL FORM IN QUADRATIC FUNCTIONS

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#### **Abstract**

Representation is an important element in mathematics learning that helps make abstract mathematical ideas more concrete. Changes between forms of representation, called translations of mathematical representations, are necessary in mathematics learning. However, students often struggle with this process, as evidenced by errors in problem-solving. These errors can be categorized into three types: interpretation errors, implementation errors, and preservation errors. Based on these facts, a solution is needed to overcome student errors. However, to formulate the right solution, an in-depth study is needed regarding student errors in translation. Therefore, this study aims to analyze and describe junior high school students' errors, especially in translating representations from symbolic to graphical form in quadratic function material. The method used is descriptive qualitative, involving three class IX students with different mathematical abilities as research subjects. The results of this study showed that highability students made few interpretation, implementation, and preservation errors from the stage of unpacking the source to constructing the target. Moderate-ability students made some interpretation and implementation errors from the stage of unpacking the source to constructing the target. Meanwhile, lowability students made many interpretation, implementation, and preservation errors from the stage of unpacking the source to determining equivalence. To overcome these errors, teachers can apply level 1 scaffolding (environmental provisions) and level 2 scaffolding (explaining, reviewing, and restructuring). In conclusion, students with different mathematical abilities each have difficulties in making translations. The factors causing these errors include inaccuracy, neglect of important aspects, inappropriate habits, and conceptual errors.

**Keywords**: Quadratic function; translation errors; translation from symbolic to graphical form; translation of mathematical representations.

#### Abstrak

Representasi termasuk elemen penting dalam pembelajaran matematika yang dapat digunakan untuk menyatakan ide-ide matematika bersifat abstrak menjadi lebih konkret. Perubahan antar bentuk representasi yang disebut translasi representasi matematis diperlukan dalam pembelajaran matematika. Namun, siswa kesulitan melakukan translasi yang dibuktikan dengan adanya kesalahan dalam menyelesaikan soal. Kesalahan-kesalahan tersebut dibedakan menjadi 3 jenis, yaitu kesalahan interpretasi, implementasi, dan preservasi. Berdasarkan fakta tersebut diperlukan adanya solusi untuk mengatasi kesalahan siswa. Akan tetapi, untuk merumuskan solusi yang tepat perlu adanya telaah yang mendalam terkait kesalahan siswa dalam melakukan translasi. Oleh karena itu, penelitian ini bertujuan untuk menganalisis dan mendeskripsikan kesalahan siswa SMP khususnya dalam melakukan translasi representasi dari simbolik ke grafik pada materi fungsi kuadrat. Metode yang digunakan yaitu kualitatif deskriptif dengan 3 siswa kelas IX yang memiliki kemampuan matematis berbeda sebagai subjek penelitian. Hasil penelitian ini menunjukkan siswa berkemampuan tinggi sedikit melakukan kesalahan interpretasi, implementasi, dan preservasi pada tahap membongkar sumber hingga tahap mengkonstruksi target. Siswa berkemampuan sedang cukup banyak melakukan kesalahan interpretasi dan implementasi pada tahap membongkar sumber hingga tahap mengkonstruksi target. Sementara itu, siswa

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berkemampuan matematis rendah banyak melakukan kesalahan interpretasi, implementasi, dan preservasi pada tahap membongkar sumber hingga tahap menentukan kesepadanan. Untuk mengatasi kesalahan-kesalahan tersebut, guru bisa menerapkan *scaffolding* level 1 (*environmental provisions*) dan *scaffolding* level 2 (*explaining, reviewing*, dan *restructuring*). Jadi dapat disimpulkan bahwa setiap siswa dengan kemampuan matematis berbeda masing-masing memiliki kesulitan dalam melakukan translasi. Faktorfaktor penyebab kesalahan tersebut yaitu ketidaktelitian, pengabaian aspek penting, keterbiasaan yang kurang tepat, dan kesalahan kosep.

Kata kunci: Fungsi kuadrat; kesalahan translasi; translasi dari bentuk simbolik ke bentuk grafik; translasi representasi matematis.



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#### **INTRODUCTION**

Representation is one of the important learning elements in mathematics (Mainali, 2021). This is because representation is key thinking, reasoning, and communicating mathematically (Niss & Højgaard, 2019). Through representation, students can more easily understand abstract mathematical ideas in a concrete way (Annajmi & Afri, 2019; Astuti, 2017). In addition, representation is also one of the standards of the school mathematics process (Allen et al., 2020). As a process standard, representation is useful for developing and improving thinking skills through the construction and abstraction of students' knowledge (Rahmawati & Hidayanto, 2017).

Changing modes of representation is necessary for learning mathematics (Mainali, 2021). The process of changing from one form of representation to another is called translation (Rahmawati & Anwar, 2020). The process of representation translation consists of four stages: unpacking the source, preliminary coordination, constructing the target, and determining equivalence (Bossé & Chandler, 2014). The ability to make these translations is crucial for understanding and performing mathematical activities. However, students often struggle with this process (Nurrahmawati et al., 2021; Swastika et al., 2018).

The difficulties experienced by students are evidenced by various errors translating mathematical sentations (Nurrahmawati et al., 2021). Adu-Gyamfi (2012) identified three types of errors students make in the translation process: implementation, interpretation, and preservation errors. Implementation errors occur when students incorrectly perform calculations, such as changing the order of coordinates in ordered pairs, forgetting to add a negative sign to a number, or failing to execute a step correctly. Interpretation errors occur when students incorrectly ascribe, describe, or exemplify attributes or properties in both the and target representations. source Meanwhile, preservation errors occur when students can maintain meaning congruence between the source and target representations for attributes or properties they identify themselves, but they often fail to ensure other relevant attributes or properties are translated correctly. This usually happens when important attributes that are identified from the source representation are not properly encoded in the target representation. One example of a preservation error is when a line graph is extended beyond the plotted points of the implementation steps.

This fact shows that there needs to be a solution in mathematics learning to overcome students' difficulties or errors DOI: <a href="https://doi.org/10.24127/ajpm.v14i2.10822">https://doi.org/10.24127/ajpm.v14i2.10822</a>

in translating mathematical representations. However, to formulate the right solution, a more in-depth study is needed regarding students' errors or difficulties in translating mathematical representations. Therefore, this study aims to analyze and describe junior high school students' errors, especially in translating mathematical representations from symbolic form to graphical form in quadratic function material. Many studies have examined the translation errors of mathematical representations, one of which is the study by Rahmawati et al. (2022). However, this study is different from the existing relevant studies. The difference lies in the type of translation used. In addition, by referring to the research results, this study also provides several solutions that can be applied to overcome student errors, especially in translating from symbolic to graphical form.

#### **METHODS**

This research is descriptive qualitative because it aims to analyze and describe representation translation errors made by students with different mathematical abilities. This research was conducted in three stages, namely preparation, implementation, completion. In the preparation stage, the researcher conducted a literature review, determined the time and location of the study, carried out a study, developed preliminary research instruments, and validated them. During the implementation stage, the researcher administered a test on translating mathematical representations from symbolic to graphical form, selected the research subjects, and conducted interviews. Finally, in the completion stage, the researcher analyzed the research results and drew conclusions.

This research was conducted at an MTs (Islamic junior high school) in Malang City during the even semester of the 2023/2024 academic year. The subjects of this study were three ninthgrade students: one with mathematical ability, one with moderate mathematical ability, and one with low mathematical ability. The selection of samples was based the on considerations of mathematics teachers, good communication skills, and the frequency of mistakes made by students in each ability group. The material used in this study was the quadratic function, as it had already been covered by ninthgrade students at MTs Al-Huda.

Data collection techniques employed both test and non-test methods. The test method involved giving test questions to all ninth-grade students to gather data on the errors made during the translation process. non-test method involved conducting interviews with selected subjects after they completed the test questions. Meanwhile, the research instruments used in this study included the main instrument, which was the researchers themselves, and supporting instruments, which consisted of one test question on translating from symbolic to graphical representations of quadratic functions and a set of interview guideline sheets.

The research instrument was validated by a lecturer from the Mathematics Department at State University of Malang before being used for data collection. The validation results were processed using the following formula and then interpreted to determine whether the instrument was valid for use in the field.

Level of validity =  $\frac{\text{Total score achieved}}{\text{Sum of the highest scores}} \times 100\%$  (1)

Based on the formula above, the test instrument and a set of interview guideline sheets were declared valid, making them suitable for use. The data analysis technique employed in this research is triangulation of sources and methods. Source triangulation is used to compare test results with interview results, while method triangulation is employed due to the use of two data collection techniques.

#### RESULTS AND DISCUSSION

Based on the results of the translation test and the mathematics teacher's considerations, three students were selected as research subjects: SRD (high mathematical ability), NSR (medium mathematical ability), and AAS (low mathematical ability). The following are the errors observed in these three students during the process of translating representations from symbolic to graphical form.

# 1. Mathematical Representation Translation Errors in the High-Ability Group (SRD)

In translating mathematical representations from symbolic to graphical, SRD made errors at the stages of unpacking the source, preliminary coordination, and constructing the target. The following are the mistakes made by SRD.

# a. Errors Made by SRD at the Source Unpacking Stage

At the source unpacking stage, SRD incorrectly identified the values of a and b from the function h(t) = $120t - 4t2h(t) = 120t - 4t^2$ . stated that a is  $-4t^2$  and b is 120t. Through the interview, SRD revealed that they believed the variables should be included in the values of a and b, and that the variables would not be included when these values substituted into the formula. This error is an interpretation error, as SRD misinterpreted the mathematical notation. As noted by Adu-Gyamfi (2012), interpretation errors occur when students incorrectly assume attributes of the source representation. Figure 1 shows SRD's error in identifying the values of a and b.

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Figure 1. SRD's errors at the source unpacking stage

b. Errors Made by SRD at the Preliminary Coordination Stage

At the preliminary coordination stage, SRD used  $x_{1,2}$  as the notation to express the quadratic formula. This indicates that SRD did not relate the

notation to the context of the problem. The correct notation should be  $t_{1,2}$ , as the formula is used to determine the time when the rocket is initially launched and the time when the rocket lands.

Based on this, SRD made an interpretation error. This is due to SRD's failure to adapt the notation or symbols according to the specific context of the problem, despite understanding the use of the quadratic formula. SRD's error at the preliminary coordination stage is shown in Figure 2.

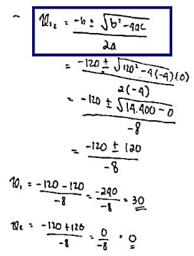


Figure 2. SRD's errors at the preliminary coordination stage

# c. Errors Made by SRD at the Target Constructing Stage

At the stage of constructing the target, SRD made three types of errors: interpretation, implementation, preservation errors. Interpretation errors occurred when SRD failed to label the horizontal axis and vertical according to the context of the problem. An implementation error occurred when SRD incorrectly wrote the ordered pair to describe the coordinate points that had been plotted, writing (0,30) instead of (30,0). This aligns with Adu-Gyamfi (2012) observation that implementation errors occur when students change the order of coordinates in ordered pairs.

Meanwhile, a preservation error occurred when SRD extended the curve past the horizontal axis intersection point. Although the graph drawn by SRD was correct, the extension of the

curve was incorrect because it did not fit the context of the problem. Figure 3 shows SRD's errors in constructing the target.

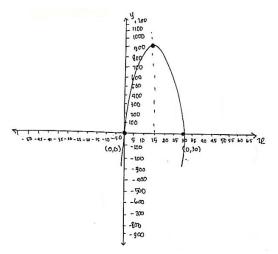


Figure 3. SRD's errors at the target constructing stage

In addition, the interview results showed that SRD was aware of the function's range but had a habit of extending the curve when drawing the graph. This habit led to the preservation error. According to Adu-Gyamfi (2012), a preservation error occurs when the line graph is extended beyond the plotted points, especially when the line passes the intersection point of the horizontal axis.

# 2. Mathematical Representation Translation Errors in the Medium-Ability Group (NSR)

NSR made several errors during the translation process from symbolic to graphical representations, from the stage of unpacking the source to the stage of constructing the target. The following are NSR's errors at each stage.

#### a. NSR's Errors at the Source Unpacking Stage

NSR incorrectly rewrote the given function and misstated the values of *b* and *c*. On the answer sheet, NSR wrote

that the function is  $h(t) = -4t^2 + 120$ and incorrectly identified a as -4, b as 0, and c as 120. This error occurred because NSR failed to correctly rewrite the function  $h(t) = 120t - 4t^2$ . NSR's mistake in rewriting the given function

was an interpretation error, as NSR did not attend to the details of the information in the problem. Figure 4 shows NSR's errors at the source unpacking stage.

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Figure 4. NSR's errors at the source unpacking stage

The interview results revealed that NSR was confused about determining the values of a, b, and c from the function  $h(t) = 120t - 4t^2$ . Initially, NSR stated that aaa was  $-4t^2$ , b was 120t, and c was 0. NSR later expressed doubts about the values of b and c. This confusion led to an interpretation error, as NSR incorrectly considered the attributes in the source representation (Adu-Gyamfi, 2012).

# b. NSR's Errors at the Preliminary Coordination Stage

At the preliminary coordination stage, when determining the initial coordinates of the rocket's launch and landing, NSR wrote "abc" as the description of the quadratic formula. This indicates that NSR did not provide a description of the quadratic formula relevant to the context of the problem. In this case, NSR made an interpretation error, as NSR incorrectly assumed an attribute (Adu-Gyamfi, 2012).

Figure 5 shows NSR's error at the preliminary coordination stage.

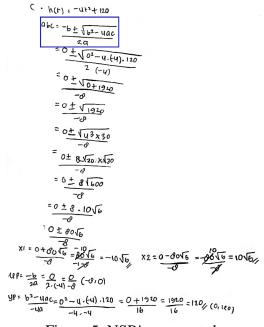


Figure 5. NSR's errors at the preliminary coordination stage

#### c. NSR's **Errors** at the Target Constructing Stage

At the target construction stage, NSR incorrectly substituted the values of bbb and ccc into the quadratic formula. This error is an implementation error because certain steps were not executed correctly. According to Adu-Gyamfi (2012) implementation

errors occur when a calculation step is completed incorrectly. This implementation error resulted from an earlier interpretation error made by NSR in determining the values of a, b, and c. The interpretation error occurred first and led to the implementation error (Adu-Gyamfi, 2012). Additionally, there was a calculation error in the quadratic formula when NSR failed to simplify the root form, which is another implementation error that led to an incorrect result for the horizontal axis intersection.

The answer also indicated that **NSR** could not determine coordinates of the horizontal axis intersection point and the coordinates of the vertex. NSR mistakenly believed that the values of  $x_p$  and  $y_p$  could produce two different coordinate points, namely  $(x_p, 0)$  and  $(0, y_p)$ . During the interview, NSR could not explain the reasoning behind writing the coordinate points in this manner. Additionally, NSR made an error in determining the division result in the  $x_p$  formula. However, NSR realized this calculation mistake after being confirmed through the interview. Based on the explanation above, NSR made an interpretation error because NSR did not fully understand how to write and interpret the vertex within the context of the coordinates.

In addition, the graph created by NSR was incorrect due to both interpretation implementation and errors. The interpretation error occurred because NSR did not information on the horizontal and vertical axes, indicating a lack of attention to important aspects of graphing. The implementation error happened when NSR failed to fully write the scale on both axes and skipped a step in drawing the graph, resulting in

a curve being drawn without plotting the coordinate points. Figures 6 and 7 illustrates NSR's errors at the target construction stage.

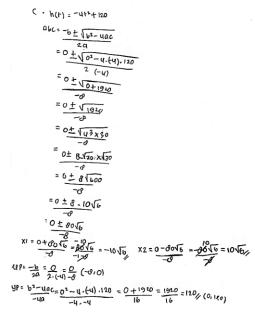


Figure 6. NSR's errors at the target construction stage: part I

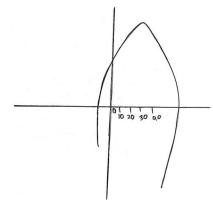


Figure 7. NSR's errors at the target construction stage: part II

### 3. Mathematical Representation Translation Errors in the Low-Ability Group (AAS)

AAS made mistakes at the stages of unpacking the source, initial coordination, constructing the target, and determining equivalence. The following are the errors made by AAS at each stage of the translation process.

a. AAS's Errors at the Source Unpacking Stage

AAS stated that  $t^2$  is -4 and t is 120. This indicates that AAS did not use the correct notation for expressing the coefficients of a second-degree

variable and a first-degree variable. This error is classified as an interpretation error. AAS's interpretation error at the source unpacking stage is shown in Figure 8.

b.) menyerasaikan hitungan itu dan Seterah Itu menggambar grafik

Figure 8. AAS's errors at the source unpacking stage

Based on the interview results, AAS did not understand the meaning of some keywords in the problem. AAS was unable to correctly identify the initial time when the rocket was launched and the height of the rocket when it fell to the ground. This error is classified as an interpretation error because it involves a misunderstanding of the attributes in the source representation.

## b. AAS's Errors at the Preliminary Coordination Stage

At the preliminary coordination stage, the formula written by AAS to determine the time for the rocket to dump fuel was incorrect. AAS failed to include a negative sign in the formula. This error is categorized as an implementation error due to AAS's failure to correctly apply the formula in a step. The implementation error made by AAS is illustrated in Figure 9.

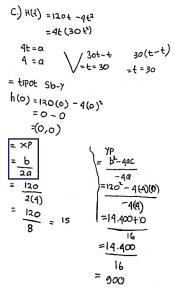


Figure 9. AAS's errors at the preliminary coordination stage

### c. AAS's Errors at the Target Construction Stage

In constructing the target, AAS made 2 implementation errors and 3 interpretation errors. The first implementation error occurred when AAS incorrectly substituted the value of aaa into the  $x_p$  formula. The second implementation error happened when AAS was unable to factorize correctly. This is classified as an implementation error because AAS failed to execute the steps needed to form the target representation.

The first interpretation occurred when AAS failed to include information on the coordinate axes. This indicates that AAS overlooked an important aspect of the target representation. The second interpretation error occurred when AAS did not write a negative sign on some of the y-axis scales below the origin, even though interview results indicated that AAS knew the scales on the y-axis below the origin were negative. The third interpretation error happened when AAS could not accurately determine or describe the coordinate points. AAS stated that the coordinate points drawn were -4 and 4, and the vertex drawn was -15. Figures 10 and 11 illustrate AAS's errors at the target construction stage.

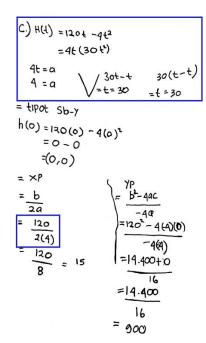


Figure 10. AAS's errors at the target construction stage: part I

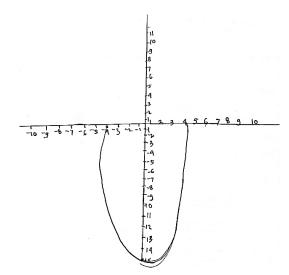


Figure 11. AAS's errors at the target construction stage: part II

d. AAS's Errors at the Stage of Determining Equivalence

In determining equivalence, AAS was unsure about the target representation created. AAS recognized that the graph should be opening downwards based on the value of aaa. However, the graph produced by AAS was an upward-opening parabola, and AAS did not correct this error. This indicates a preservation error, as AAS maintained the incorrect graph despite evidence that the shape was wrong. Figure 12 illustrates this preservation error.

e tidak sesuai karena saya merasa ada salah hitung dan saya salah menggambar gravik karena seharusnya gambarnya gravik terbuka ke bawah

Figure 12. AAS's errors at the determining equivalence stage

Based on the study results, the most common types of errors made by students with high, medium, and low mathematical abilities when translating symbolic representations to graphs were

interpretation and implementation errors. Preservation errors were observed only in students with high and low mathematical abilities.

Interpretation errors occurred when students incorrectly rewrote the source representation, misinterpreted the attributes in the source representation, failed to understand the meaning of keywords, did not properly name or describe elements according to the problem context in the quadratic formula, did not label the axes appropriately, and wrote coordinate points incorrectly. **Implementation** errors included incorrect formula application, calculation errors, incorrect ordering of pairs, and skipping steps in constructing the representation. Preservation errors happened when students extended the curve beyond the horizontal axis intersection point, resulting in a graph that did not align with the function specified in the problem. range Additionally, preservation errors occurred when students retained an incorrect graph shape despite realizing that it did not match the expected shape based on the coefficient of the squared variable in the quadratic function.

One way to address student errors is by providing scaffolding (Priyati & Mampouw, 2018), as it can help minimize the mistakes students make (Purwasih & Rahmadhani, 2022). To reduce the likelihood of interpretation, implementation, preservation and errors, teachers can employ level 1 scaffolding (environmental provisions) and level 2 scaffolding (explaining, reviewing, and restructuring). Level 1 scaffolding includes environmental provisions such as group learning (Anghileri, 2006; Utomo & Santoso, 2021). Group learning encourages collaboration between students, where each member provides support,

motivation, and shares knowledge to jointly solve mathematical problems in order to achieve learning goals (Berta & Hoffmann, 2020). In addition, through group learning activities, students can share ideas with group members during discussions, which can indirectly enhance their creativity in understanding mathematical concepts and solving problems, thereby positively impacting learning outcomes (Nasution & Surya, 2017).

Level 2 scaffolding involves explaining, reviewing, and restructuring. Teachers can explain the meaning of problems to help students generate solutions (Rahayuningsih & Qohar, 2014). Reviewing involves asking questions to follow-up encourage students to revisit the given questions in better understand order to (Syahraini et al., 2023). Rahayuningsih & Qohar (2014) also suggest that reviewing can be enhanced providing additional examples to help students understand better. As noted by Susilo (2019) this can include providing prompting and probing questions. Meanwhile, restructuring involves a question and answer process to help students find the correct answers (Loka & Fuad, 2023).

In contrast to research by Rahmawati et al. (2022), which examined the processes and types of student errors in translating representations from graphic to symbolic, table to symbolic, and verbal to symbolic forms in quadratic function material at the high school level, this study reveals that most junior high school students make errors when translating representations symbolic to graphic Accordingly, this study offers additional contribution by reinforcing previous findings that similar types of interpretation errors namely,

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implementation errors are also encountered by junior high school students, albeit in different forms of representation translation. Furthermore, the study indicates that such errors are not limited to low-ability students, but are also found among students with medium and high levels of mathematical ability.

In addition to providing insights into students' errors in translating symbolic representations into graphical form, this study also offers appropriate solutions to address these errors. However, the study is limited to examining students' errors in a single type of translation and material. Despite this limitation, the study is expected to serve as a relevant reference for future research on the same topic. Additionally, solutions the addressing mathematical representation translation errors described in this study can be used by teachers and researchers to improve students' skills in translating mathematical representations.

# CONCLUSIONS AND SUGGESTIONS

Based on the result of the research, it can be concluded that students with varying levels of mathematical ability each face difficulties when translating representations from symbolic to graphical form. This is evidenced by the errors occurring at each stage of translation during the process of solving quadratic function problems. These errors led to inaccuracies in forming the target representation.

Students with high and low mathematical ability made errors in interpretation, implementation, and preservation. However, students with high mathematical ability could form target representations correctly and made fewer errors than students with low mathematical ability. Meanwhile, students with moderate mathematical ability made a large number of errors in interpretation and implementation. These errors were caused by several factors, including inaccuracy, neglect of important aspects especially in target representations, inappropriate habits in making target representations, and conceptual errors.

Based on the limitations of the study, further research is needed to explore other types of representation translation errors and other materials. Future research should focus on developing learning strategies that can improve students' ability to translate mathematical representations. In addition, teachers should apply Level 1 (environmental provisions) and Level 2 (explaining, reviewing, and restructuring) scaffolding to minimize translation process errors.

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