

## PRE-SERVICE TEACHERS' SYMBOLIC-VISUAL REPRESENTATION SKILLS IN ADDING AND SUBTRACTING FRACTIONS

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### Abstract

Understanding the concept of fractions is an important component of knowledge regarding the representation of fractions which is considered a structured problem in educational mathematics today, because the underlying concepts are considered very complex, partly due to different interpretation and representation skills related to the concept of number operation procedures. The current study attempted to describe pre-service teachers' symbolic-visual representation skills in adding and subtracting fractions. It specifically aimed to analyze the teachers' ability to construct either a symbolic or visual representation in fraction operations. A case study was conducted by involving 17 fifth-semester students who were enrolled in a teacher training course at one of University in Ternate, Indonesia. The participants consisted of two males and fifteen females. The study data were gathered using a fraction problem-solving test. The results showed that: 1) the majority of participants succeeded in performing fraction operations and making representations by adding and subtracting fractions; 2) the participants demonstrated proficiency in executing fraction operations but showed poor ability in generating different representations of fractions; 3) the participants were more successful in creating visual representation for fraction addition compared to fraction subtraction. The participants also excelled in constructing circle models as visual representations, outperforming other visual representation models such as number lines, rectangular models for transition situations, and numerical representations of mixed fractions.

**Keywords:** Addition and subtraction of fractions; representation making skills; understanding of representations.

### Abstrak

Pemahaman konsep pecahan merupakan komponen pengetahuan yang penting mengenai representasi pecahan dianggap sebagai masalah terstruktur dalam pendidikan matematika saat ini, karena konsep yang mendasarinya dianggap sangat kompleks, sebagian karena ketrampilan interpretasi dan representasi yang berbeda terkait konsep prosedur operasi bilangan tersebut. Penelitian ini bertujuan untuk mendeskripsikan keterampilan representasi simbolik-visual guru prajabatan dalam menjumlahkan dan mengurangi pecahan. Penelitian ini secara khusus bertujuan untuk menganalisis kemampuan guru dalam mengkonstruksi representasi simbolik maupun visual dalam operasi pecahan. Studi kasus dilakukan dengan melibatkan 17 mahasiswa semester lima yang mengikuti kursus pelatihan guru di salah satu universitas di Ternate, Indonesia. Pesertanya terdiri dari dua laki-laki dan lima belas perempuan. Data penelitian dikumpulkan dengan menggunakan tes pemecahan masalah pecahan. Hasil penelitian menunjukkan bahwa: 1) sebagian besar peserta berhasil melakukan operasi pecahan dan membuat representasi penjumlahan dan pengurangan pecahan; 2) para peserta menunjukkan kemahiran dalam melaksanakan operasi pecahan tetapi menunjukkan kemampuan yang buruk dalam menghasilkan representasi pecahan yang berbeda; 3) peserta lebih berhasil membuat representasi visual penjumlahan pecahan dibandingkan pengurangan pecahan. Para peserta juga unggul dalam menyusun model lingkaran sebagai representasi visual, mengungguli model representasi visual lainnya seperti garis bilangan, model persegi panjang untuk situasi transisi, dan representasi numerik pecahan campuran.

**Kata kunci:** penjumlahan dan pengurangan pecahan; keterampilan membuat representasi; pemahaman representasi.



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## INTRODUCTION

Representation is the act of conveying mathematical ideas by different means, such as visual images, concrete objects, tables, graphs, numerical and alphabetical symbols, spreadsheet presentations, and so forth. Mathematical representation is the basis for someone to understand and use ideas in solving problems (NCTM, 2000). Using several representations during problem solving allows students to approach the subject from various perspectives and explore it thoroughly (Abrahamson et al., 2020; Barbosa & Vale, 2021; Coles & Sinclair, 2019). This, in turn, is the beneficiary of a strong conceptual understanding. Multiple representations are a learning theory in mathematics that aids in the development of a more profound and adaptable comprehension of concepts and processes among students (Hiebert et al., 1997; Österman Tove & Kajsa, 2019). The need to use appropriate representation in learning mathematics aims to ensure that someone has a complete understanding of mathematical concepts and is able to express mathematical ideas (Castro et al., 2022; Duval, 2006). Moreover, using different representations in teaching mathematical concepts and making transitions between forms of representation is very important in terms of complete internalization of mathematics (Hiebert et al., 1997; NCTM, 2020). Therefore, the use of representations has become an important topic in mathematics learning for the past three decades in school mathematics standards to develop students' abilities to use appropriate representations and to make correct translations of different representations (NCTM, 2020).

Past studies have demonstrated

that using representations aids students in comprehending and enhancing their problem-solving skills (Barbosa & Vale, 2021; Perdomo-Díaz et al., 2017; Xie & Masingila, 2017). If the ability to move between alternative representations is not feasible, then it might be concluded that mathematics cannot be comprehended at a conceptual level (Bicer, 2021; Chang et al., 2021; Crooks & Alibali, 2014). Research on multiple representations has also shown that students' proficiency in transitioning between different representations is contingent upon their problem-solving skills, proficiency in algebraic expressions, and understanding of functions (Behr et al., 1983; Martín-Fernández et al., 2019).

However, there has been no detailed examination of the operations of adding and subtracting fractions using different representations. In this context, there are unanswered questions about the quality of creating and using multiple representations in mathematics. Therefore, this research was performed to demonstrate pre-service teachers' performance in transition situations among representations (numerical, number line, circle model, and rectangular model) given in addition-subtraction operations on fractions. The current study also analyzed the teachers' ability in constructing every type of representation.

Reform in the domain of mathematics education is deeply grounded in the pursuit of methods that enable pupils to acquire mathematical knowledge and skills (Ernest et al., 2016; Simon, 2017). Engaging in mathematics learning entails more than just manipulating mathematical symbols. It requires the coordination

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and interpretation of mathematical relationships and situations using special language, symbols, graphics, or other representations. Additionally, it involves clarifying problems, minimizing consequences, and developing suitable tools (Abrahamson et al., 2020; Coles & Sinclair, 2019). A growing theoretical perspective on mathematics learning suggests that employing various representations to link graphical, tabular, symbolic, and verbal descriptions of mathematical relationships and problem situations during instruction can both empower students and support their development of an understanding of mathematical relationships and concepts (Hiebert et al., 1997; NCTM, 2020). Furthermore Vale & Barbosa, (2018) the right vision for solving problems, among other things, engaging with diverse problems that allow for the application of different thought processes, including multiple representations and various solution strategies, can be beneficial.

Some literature suggests that mathematics instruction needs to improve students' skills in transitioning between different representations to support conceptual learning (Marks et al., 2021; Polotskaia et al., 2022; Sokolowski, 2018). Expressed as mathematical language, representation in general is the process of modeling abstract concepts and symbols concretely in the real world (Kaput et al., 1985). Through representation, students can learn mathematics from various aspects (Mainali, 2021). Using multiple representations equips students for more advanced mathematical concepts, while also offering them a variety of problem-solving approaches and reinforcing their conceptual understanding (Gilmore et al., 2017; Rittle-Johnson et

al., 2015). Multiple representations can be described as the process of making abstract concepts or symbols more tangible and relatable to everyday life, while also defining the connections between objects or symbols in mathematics (Kaput et al., 1985).

Learning environments that utilize multiple representations contribute to students' conceptual understanding of mathematical identity (Gitonga, 2016; Pedersen et al., 2021). In this context, it is important for mathematics teachers to consider and skillfully incorporate multiple representations of information whether verbal, numerical, visual, or graphical by leveraging advancements in technology, instead of relying solely on verbal and mathematical language. Fractions, in particular, offer an opportunity for teachers to utilize diverse representations. From elementary school onwards, students encounter fractions alongside natural numbers, whole numbers, and rational numbers. Challenges faced by many students in understanding fraction concepts and operations have been widely recognized (Amo-Asante & Bonyah, 2023; Copur-Gencturk, 2021; Olanoff et al., 2014). Utilizing multiple representations can effectively improve student learning (Rau, 2017b; Rau & Matthews, 2017). Different representations highlight complementary conceptual aspects of a topic. For instance, when learning about fractions, students often encounter visual representations: a circle demonstrates fractions as parts of a whole, whereas a number line emphasizes fractions as measures of relative magnitude (Behr et al., 1983; Kieren, 1976).

In the field of mathematics education, global research on students' understanding of fractions and fraction operations indicates that learning

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fractions is a challenging and complex process. Obstacles to students' reasoning about fractions often stem from factors such as an early focus on whole numbers (e.g. Behr et al., 1983) and inadequate ability to convert between multiple representations (Tunç-Pekkan, 2015). Conversely, it is argued that while students may be able to perform fraction algorithms with ease, they often lack a true understanding of the underlying meaning of these algorithms (Copur-Gencturk, 2021; Siegler & Lortie-Forgues, 2015; Spitzer et al., 2024; Webel & DeLeeuw, 2016; Simon et al., 2018). Given the challenges and misconceptions students face when adding fractions, employing multiple representations in teaching is intended to address these difficulties and enhance students' conceptual understanding of fractions. This approach aims to help students grasp the meaning of fractions more effectively (Tunç-Pekkan, 2015).

Research on fractions typically centers around the challenges students face when performing operations with fractions and the development of various representations of these fractions (Kar & Işık, 2014), the representation preferences that students use in fractions (Kara & İncikabı, 2018), and misconceptions they have about fractions (Bentley & Bossé, 2018; Lee & Boyadzhiev, 2020). However, there has been no research that examines the preferences of student teachers, the situation of achievement in their preferences and the situation of creating representations. The development of the way students think about representations in mathematics learning has revealed the need to use multiple representations (Alkhateeb, 2018; Kang

& Liu, 2018). Apart from that, different representations also play an important role in building strong learning beliefs at different educational levels (Dreher et al., 2016).

To strike a balance between conceptual understanding and algorithmic knowledge, it is essential for both students and teachers to interchange information among different representations of fractions and engage in operations involving fractions (Resnick et al., 2023; Sidney & Alibali, 2017). However, Bentley & Bossé, (2018) have investigated students' work and communication regarding fractions and arithmetic operations on fractions is still weak. Several studies have demonstrated teachers' weaknesses in integrating multiple representations in their teaching environment (Bautista et al., 2019). Given that the incorporation and emphasis on multiple representations are integral to mathematics teaching programs in Indonesia, it is crucial for both current and future teachers to be knowledgeable about and proficient in utilizing these representations. In this context, there are unanswered questions about the ability of pre-service teachers to create and utilize multiple representations, and this research question is worth answering. Once again, the findings of this study are significant for enhancing classroom teaching practices. Additionally, the research outcomes are expected to contribute to the exploration of alternative pedagogical methods for mathematics educators and researchers, as well as to examine the impact of these methods on student teachers' representational skills during classroom instruction.

#### Research Questions

1. How do pre-service teachers

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- perform in creating multiple representations in adding and subtracting fractions?
2. How do these teachers construct multiple representations using addition and subtraction operations of fractions?

## METHODS

This study is categorized as case study research, which focuses on the performance of pre-service teachers as they transition between various representations of adding and subtracting fractions. The case study approach enables researchers to concentrate on a specific situation whether it be an event, individual, or group and explore the cause-and-effect relationships in greater detail using the gathered data (Creswell, 2014).

### *Study Group*

The study group participating in this research consisted of 17 (seventeen) fifth semester students at one of the universities in Ternate, North Maluku, Indonesia. No one in this group had previous teaching experience. Criteria-based purposive sampling technique was applied in forming the study group. Of the 17 students who agreed to participate in the case study, 2 (two) were men, and 15 (fifteen) were women. All of them were subjects to analysis in this paper. The selection criteria for the research participants included that all students were in semester 5 and had successfully completed courses related to mathematics education, such as Basic Mathematics Teaching Methods I and Basic Mathematics Teaching Methods II.

### *Data Collection and Analysis*

Two data collection tools were used in this study. The first tool was fraction problem-solving tasks used in

the preliminary study (observation). The aim of administering the tasks to the participants was to review the basic operating strategies used by the participants when working on fraction operations. Preliminary analysis of the study told us the difficulties and strategies used by the participants to solve problems taken from the context of a particular passage. The result of the problem-solving task would outline the problems from the participants' problem-solving strategy. The second data collection tool was "Multiple Representations in Fraction Operations Test". The validity and reliability of the test was determined based on the opinion of experts from the field of mathematics education. The validity of the instrument was achieved through the step where experts indicated whether each test item met the requirements by marking the options "suitable", "not suitable" and "needs to be changed" according to their clarity, quality, and level.

As can be seen in Table 1, different types of fractions and fraction operations were used; like the denominator, unlike the denominator, problems with adding and subtracting fractions. According to the National education standards, the strategies needed to solve fraction problems are discussed before graduating from high school. Thus, participants' solutions to solving the fraction problems were analyzed to investigate their level of understanding of the core concepts (Ananda & Rafida, 2017). Our goal in collecting qualitative data was to add insights and provide additional description and context found in participants' fraction problem-solving scores and incorrect responses (Abdullah, 2014; Kara & Incikabi, 2018).

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*Coding Procedure*

The coding procedure in this research consisted of creating a coding list, examining relevant literature, determining the type of representation

in addition-subtraction operations in fractions. Table 2 contains the criteria used to assess the representation skills of research participants.

Table 1. Multiple Representations in Fraction Operations Test

Task	Question No.		Type of representation needed for transition
	Addition	Subtraction	
$\frac{a}{b} + \frac{c}{d}$ , and $\frac{a}{b} - \frac{c}{d}$	1	1b	Numerical
	a		Number line
	2	2b	Numerical
	a		Circular
$m\frac{a}{b} + \frac{c}{d}$ , and $n\frac{a}{b} - \frac{c}{d}$	3	3b	Numerical
	a		Rectangular
	5		Numerical
			Visual

Table 2. Criteria for creating multiple representations

Concept of Fraction	Representation		
	Numerical	Number Line	Model (circular and rectangular)
Addition and Subtraction of Fractions	1. Write the mathematical expression of the representation.	1. Construct part-whole units in accordance with operational results.	1. Construct a part-whole model unit that is appropriate to the results of the operation.
	2. Express the given operation numerically	2. Express the results of operations on a number line (showing the numerator and denominator of the results of operations with arrows or other forms of signs)	2. Express the shading of the requested model part according to the results of the operation (determining the numerator)
	3. Balancing the numerator and denominator, and/or changing the form of a mixed fraction into an improper fraction or vice versa	3. Determine the suitability of the numerator and denominator if necessary.	3. Determine the suitability of the numerator and denominator, if necessary, in the model
	4. Determine the results of the operation, and create a visual representation if necessary	4. Display the results of the number line drawing	4. Display the model

(Adapted from Kara & İncikabı, 2018)

**RESULTS AND DISCUSSION**

Table 3 reveals the participants' transition skills in creating representations for addition and subtraction of fraction operations. It was found that research participants had an average success rate in solving representation problems of 34.58% (low). Most participants succeeded in solving fraction representation problems in pure numerical transition format or using traditional standard algorithms (24.92 %).

Meanwhile, the mean score for visual representation skills was 9.66% (very low), as shown in Table 4. This result suggested that the pre-service teachers succeeded in carrying out fraction operations adequately without having a conceptual understanding in making visual representations. Therefore, the overall success of some students was not supported by more than half of the other students, who experienced 65.42% (high) misconceptions in terms of numerical

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procedures and visual representation skills (Table 3).

Table 3. Participants' Fraction Representation Ability

Subject Category	Number of Subjects	Score (%)	Success Rate
A	1	80 < P < 100	Very high
B	5	60 < P < 80	High
C	1	40 < P < 60	Moderate
D	3	20 < P < 40	Low
E	7	0 < P < 20	Very low
Average		34.58	Low

(Ananda & Rafida, 2017).

Participants faced some errors and misunderstandings in the fraction operations, especially in creating visual representations. These problems were then analyzed and presented according

to predetermined criteria. The participants' scores were determined based on their responses to the questions regarding creating the respective representations (Table 4).

Table 4. Participants' Skills and Misunderstandings in Creating Visual Representations in Fraction Operations

Fraction	Stages	Percentage			
		Understanding of Operation Procedure	Skill in Creating Visual Representations	Success Rate	Misunderstanding
Improper Fractions	Number Line	29.41	10.08	39.49	60.51
	Circle	21.14	14.28	35.4	64.6
	Rectanguler	28.57	14.28	42.85	57.15
Mixed Fractions	Numerical	20.59	-	20.59	79.41
Mean		24.92	9.66	34.58	65.42

The problem-solving instrument (tasks) contained instructions on how to create and determine the representation of fraction symbols in visual form. However, most students had misunderstandings and experienced difficulties in determining the results of fraction operations. Participants' difficulties in creating visual representations are described as follows.

1. *Fraction problems in operating procedures and the formation of number line representations.*

Table 5 lists the difficulties the research participants faced in model representation during addition and subtraction of fractions. Table 5 shows

that the participants encountered problems in all the steps that must be followed during the process of forming a representation on a number line.

Table 5 presents the errors made by the participants when adding and subtracting fractions on the number line. It has been determined that the pre-service teachers made many errors in expressing the operation of adding fractions. These include errors in determining the numerator and denominator, operating the numerator and denominator, determining the result of a given operation on a number line and showing the result (with arrows or other signs), determining part-whole units on the number line, and translating

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the corresponding part-whole units on the number line image. On the other hand, it has been observed that most of

the pre-service teachers left these questions unanswered.

Table 5. Misunderstanding in fraction operation procedures and the formation of number line representations.

No	Step	Addition	Subtraction	Lots of Misunderstandings	Percentage
1	Write mathematical expressions from representations	2	1	3	8.82
2	Apply procedures for determining the numerator and denominator of fractions	2	1	3	8.82
3	Balancing the numerator and denominator, and/or changing the form of a mixed fraction into an improper fraction or vice versa	2	2	4	11.76
4	Determining the results of the operation	12	12	24	70.59
5	Determine the numerator and denominator on a number line drawing	12	12	24	70.59
6	Determining equality numerator and denominator if necessary	12	12	24	70.59
7	Translating the suitability of number line drawings	12	12	24	70.59
Amount				106	44.54
8	Not answered (no stage description)	19	19	38	15.97
Total Representation Comprehension Errors					60.51

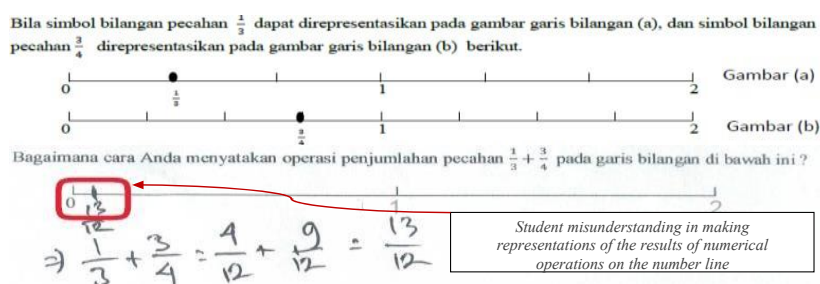


Figure 1. Errors in determining the numerator and denominator of the number line representation (SB1)

Figure 1 provides an example that pre-service teachers were more concerned with procedural skills in operating fractions. However, one of the common mistakes that these teachers frequently made when creating representations of fractions on a number line was not dividing the whole into equal parts and identifying the fraction symbol on the number line, namely placing the numerator and denominator. While the whole must be

divided into twelve equal parts on a number line, participants placed a symbolic representation of the fraction obtained. We also noted that the pre-service teachers demonstrated a strong grasp of the procedural aspects of fraction operations, as opposed to comprehending the underlying meaning of these procedures while constructing fraction representations on a number line.

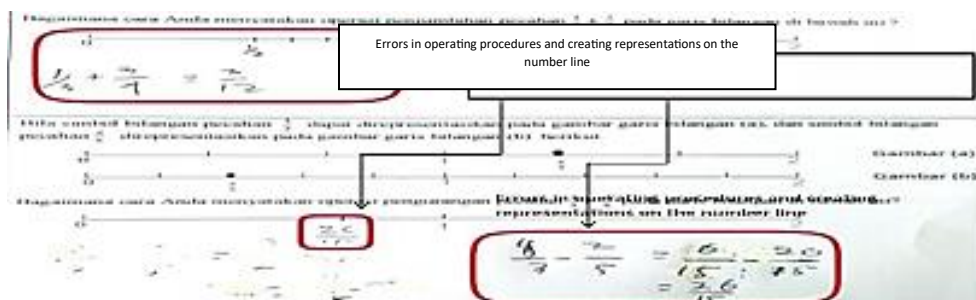


Figure 2. Errors in operating procedures and creating representations on a number line (SD1)

Figure 2 shows an error made by a pre-service teacher when performing a fraction operation. When carrying out the operation of adding fractions, the teacher only determined the denominator, did not change the numerator, and added the fractions directly, such as in  $\frac{1}{3} + \frac{3}{4} = \frac{3}{12}$ . This finding indicated the pre-service teacher's misconceptions in the basic operation of fractions, which is  $\frac{a}{b} + \frac{c}{d} = \frac{ad+bc}{bd}$ . Similarly, the teacher made an error in subtracting fractions, where the teacher should apply the basic operation of fractions, which is  $\frac{a}{b} - \frac{c}{d} = \frac{ad-bc}{bd}$ , instead of solving the fraction as in is  $\frac{4}{3} - \frac{2}{5} = \frac{6}{15} - \frac{20}{15} = \frac{26}{15}$ . These empirical facts clearly indicated that the pre-service teachers involved in this research did not understand fraction operation procedures. They were also unable to construct fraction representations on a number line.

## 2. Problems in performing fraction operations and creating model representations.

### 2.1. Problems in performing fraction operations and creating the circular model representation.

Circular representation is a representation model preferred by Subjects A and B but became a problem for other categories of subjects. Table 6

outlines the difficulties faced by pre-service teachers in adding and subtracting fractions and creating circle representations.

When adding fractions, pre-service teachers exhibited a higher tendency to commit errors during the denominator calculation and circular representation formation. In subtraction operations, it has been observed that these teachers left more questions unanswered. In addition, it is evident that the teachers committed more errors during the fraction operation stages and when determining the numerator in subtraction operations.

Figure 3 provides an example of errors that a pre-service teacher made when determining the denominator in the addition operation. According to the results from the model representation created, the pre-service teacher represented 8 units (the denominator) and marked 7 units (the numerator) in the model, concluding that the solution was  $\frac{7}{8}$ . However, when carrying out the addition operation on the model representation, the teacher appeared to overlook the denominator in the final answer and focused solely on adding the shaded areas. When faced with different circular models such as the problem of subtracting fractions, the pre-service teachers committed errors in creating a circular representation. This finding showed that pre-service teachers did not quite understand the meaning of fraction

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operations when represented in a circular model. Figure 4 suggests that the pre-service teacher applied the same procedure for adding whole numbers to fractions. This teacher performed the operation  $\frac{3}{8} + \frac{1}{2} = \frac{4}{10}$ , drew 10 units as the denominator and marked 4 units as

the numerator, hence concluded  $\frac{4}{10}$  as the solution. When the teacher performed the addition operation in a circular representation, he shaded  $\frac{4}{10}$ , which did not answer the question correctly.

Table 6. Problems faced by pre-service teachers in fraction operation procedures and the formation of circular representations.

No	Stage	Addition	Subtraction	Number of errors	Percentage
1	Write mathematical expressions from representations	5	3	8	23.53
2	Apply the procedure for determining the numerator and denominator of a fraction	5	3	8	23.53
3	Balance the numerator and denominator, and/or changing the form of a mixed fraction into an improper fraction or vice versa	6	4	10	29.41
4	Determine the results of the operation	7	7	14	41.18
5	Determine the numerator and denominator of the operation results on a circle image	9	9	18	52.94
6	Determine the equality of numerator and denominator when necessary	9	10	19	55.88
7	Translate the suitability of the circle image	9	10	19	55.88
Amount				96	40.3
8	Unanswered (No stages mentioned)	27	31	58	24.3
Total Errors in Understanding Representations					64.6

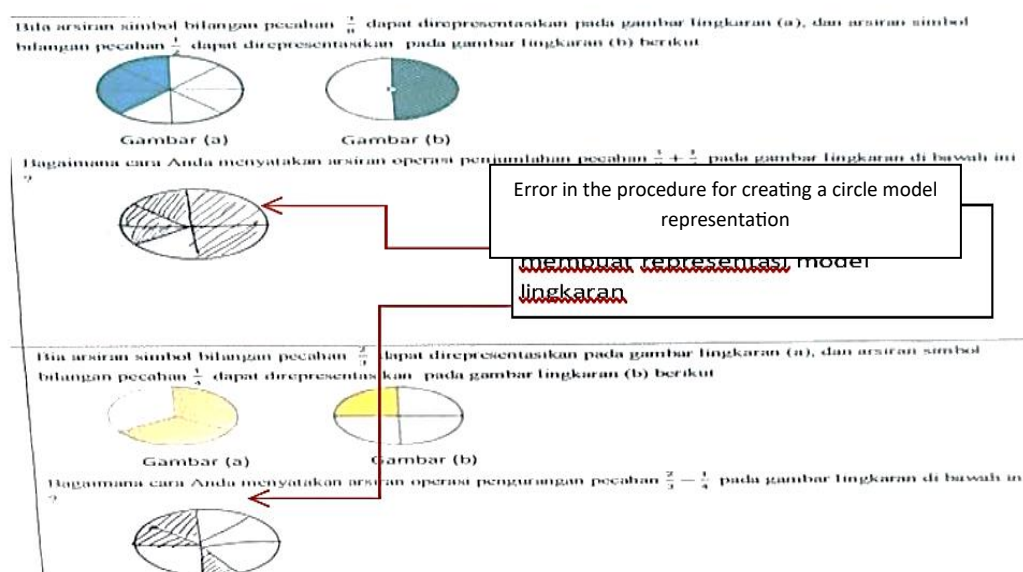


Figure 3. Errors in determining the denominator in a circular representation (SE7).

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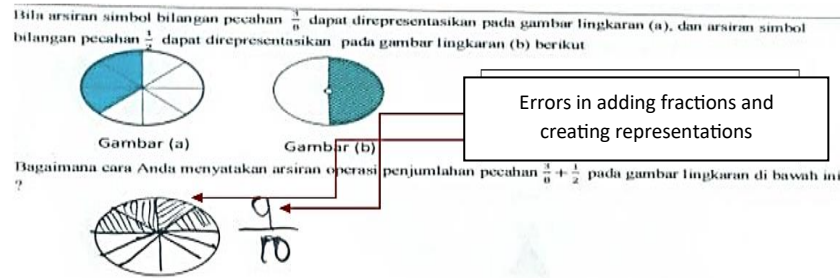


Figure 4. Errors in operating fractions resulting in an incorrect answer to the question (SD2)

## 2.2 Problems in performing fraction operations and creating rectangular representations.

Table 7 describes the challenges encountered by pre-service teachers when performing addition and subtraction operations of fractions, as well as when creating rectangular representations. Table 7 details the difficulties faced by the teachers in every step of the fraction operations and in generating rectangular representations.

When adding fractions, pre-service teachers made many mistakes in determining the denominator and fulfilling the operations to form a rectangular representation. It has been

observed that these teachers left more questions unanswered in subtraction operations. Apart from that, it can be seen that the errors made by the teachers in fulfilling the operation and determining the numerator also occurred more often in subtraction operations.

Figure 5 presents an example of an error in equalizing denominators and performing transition operations in creating a rectangular model representation. The representation of a rectangular model that is more than one unit length is a representation model that was considered difficult by all pre-service teachers.

Table 7. Problems faced by pre-service teachers in fraction operation procedures and the formation of rectangular representations.

No	Stage	Addition	Subtraction	Number of errors	Percentage
1	Write mathematical expressions from representations	2	6	8	23.53
2	Apply the procedure for determining the numerator and denominator of a fraction	2	6	8	23.53
3	Balance the numerator and denominator, and/or changing the form of a mixed fraction into an improper fraction or vice versa	3	7	10	26.47
4	Determine the results of the operation	8	6	14	41.18
5	Determine the numerator and denominator of the operation results on a rectangular image	10	7	17	50.00
6	Determine the equality of numerator and denominator when necessary	10	8	18	52.94
7	Translate the suitability of the rectangular image	10	8	18	52.94
	Amount			93	39.08
8	Unanswered (No stages mentioned)	21	22	43	18.07
	Total Errors in Understanding Representations			135	57.15

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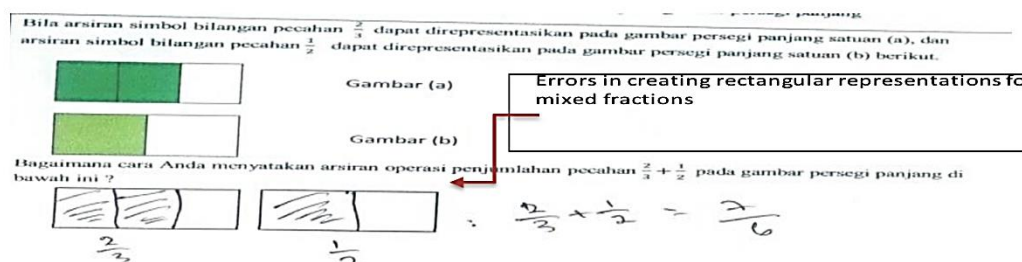


Figure 5. Errors in determining the denominator in rectangular representations (SA).

Figure 5 demonstrates that while the process for manipulating fractions was accurate, the pre-service teacher encountered difficulties when describing the outcomes of fraction operations in a rectangular model that includes several units of length. Simply put, the teacher was unable to manipulate the rectangular representations. The teacher established the equality of parts by dividing the rectangular unit into six equal parts to determine the denominator and numerator. Additionally, it is evident that the teacher possessed a comprehension of the procedures involved in the addition and subtraction operations of fractions. However, the teacher struggled with comprehending the significance of the rectangular representation model.

Figure 6 illustrates errors made by a pre-service teacher when working with fractions and constructing rectangular representations. In Figure 6, the pre-service teacher encountered difficulties in performing operations involving fractions, specifically in calculating the numerator and denominator. The teacher understood the procedures of adding and subtracting fractions in a similar manner to adding integers, where  $a + b = c$ . The same thing happened when the teacher tried a rectangular representation by adding the shaded parts. It is evident that the teacher lacked comprehension of the operational procedures for fractions and the significance of representing fractions in a rectangular model.

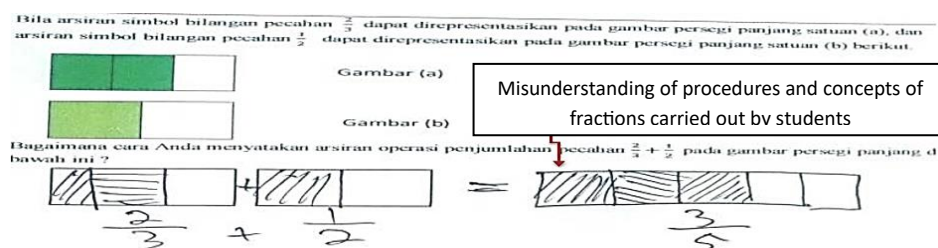


Figure 6. Errors in performing fraction operations and creating rectangular representations (SE1)

### 3. Problems in Performing Numerical Fraction Operations

Most of the pre-service teachers successfully solved fraction representation problems in pure numerical transition format or using traditional standard algorithms, which

outperformed their visual representation skills. However, Table 8 shows several problems faced by these teachers in numerical fraction operations, namely operating fractions, changing (simplifying) mixed fractions into improper fractions, determining the

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numerator and denominator, and determining the results of operations by providing other examples (other methods).

When adding fractions, the pre-service teachers made more errors in determining the numerator of the fraction, determining the denominator,

and fulfilling the operation. It was observed that these teachers left more questions unanswered in subtraction operations. Apart from that, the errors made by the teachers in fulfilling the operation and determining the numerator also occurred more often in subtraction operations.

Table 8. Problems encountered by pre-service teachers in numerical fraction operations.

No	Stages	Addition	Subtraction	Number of Errors	Percentage
1	Write numerical expressions from representations and make another example	11	7	18	52.94
2	Apply operating procedures numerically	9	8	17	50.00
3	Convert mixed fractions to improper fractions	9	9	17	52.94
4	Determine and balance the numerator and denominator	12	10	22	64.70
5	Determine the results of operations, and create representations if necessary	11	10	21	61.76
Amount				94	56.47
6	Unanswered (No stages mentioned)	14	25	39	22.94
Errors in understanding representations				138	79.41

Figure 7 provides an example of pre-service teacher's errors in performing numerical fraction operations. It can be seen that a pre-service teacher (SB2) made an error in determining the fraction numerator  $2\frac{1}{6} = \frac{2}{6}$ . The teacher also showed a misunderstanding in solving  $m\frac{a}{b} = \frac{m(b)+a}{b}$ . In addition to that, the teacher made an error in multiplying  $a(b) = ab$ . Furthermore, Figure 8 illustrates an

error made by a pre-service teacher in performing a numerical fraction operation and in creating a rectangular representation. The teacher (SE3) could successfully generate a representation model for  $\frac{1}{3}$  but made an error in creating the representation for  $2\frac{1}{6}$  and in determining the numerator of the fraction. As a result, the teacher answered the question incorrectly.

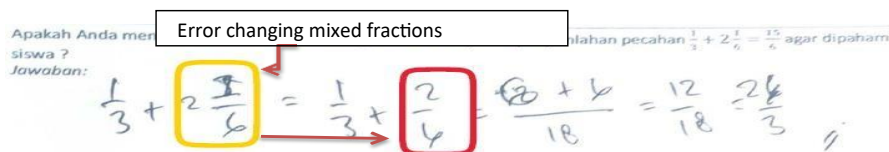


Figure 7. Errors in performing numerical fraction operations (SB2)

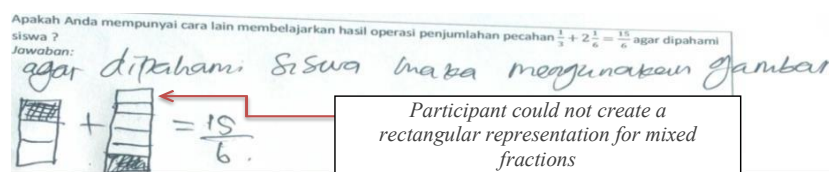


Figure 8. Errors in performing numerical procedures for mixed fractions and creating rectangular model representations carried out by prospective teacher students (SE3)

The current study aimed to assess the performance of pre-service teachers in utilizing multiple representations and transitions when working with model representations, numerical representations in adding and subtracting fractions. According to the research findings, it was concluded that pre-service teachers had relatively low success rate (34.58%) in solving fraction operations and generating representations of the fractions. The results of this study also indicated that pre-service teachers had a diminished capacity to generate multiple representations compared to undertake fraction operation procedures. In comparison to adding fractions, pre-service teachers exhibit better skills in subtracting fractions. These study findings contradict those of past studies who demonstrated a high level of effectiveness among pre-service teachers in generating multiple representation models (Abdullah, 2014; Thurtell et al., 2019).

It was also found that pre-service teachers failed to represent fractions in other models such as number lines and rectangles. In the number line representation, the difficulties experienced by these teachers in dividing the number line into the required denominators were the result of the division into missing or more equal parts, whereas the same is true for the rectangular model representation. Similar difficulties experienced by pre-service teachers

regarding number lines and rectangular models have also been demonstrated by (Abdullah, 2014; Kolar et al., 2018). In this case, where pre-service teachers preferred whole numbers that cannot be divided into parts, the situation was to consider the question as a rational number rather than a fractional meaning.

Pre-service teachers quite liked the procedures for operating fractions and creating circle model representations, although many still did not answer. This is in line with other findings which revealed high student success with numerical representations and this model (Abdullah, 2014; Kara & Incikabi, 2018; Lee & Lee, 2023). The greater success of pre-service teachers in symbolic representation can be linked to the continued use of traditional teaching methods in the classroom. Algebraic (symbolic) representation aligns well with these methods and is more prominently emphasized within this approach. In fact (Abdullah, 2014; Kara & Incikabi, 2018; Lee & Lee, 2023) obtained findings that support the idea mentioned above.

Pre-service teachers exhibited a higher frequency of errors when identifying the denominator and numerator during addition operations. Conversely, mistakes related to fraction subtraction were more prevalent during the operation itself and in determining the numerator. This pattern highlights the challenges faced by pre-service

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teachers in developing a solid understanding of the concept of fractions. Scholars have suggested the use of basic model representations in teaching fractions for the development of fraction numbers (Behr et al., 1983; Lee & Lee, 2019). These methods made the teachers rely on memorizing procedures rather than understanding conceptual procedures. This situation is in line with those in previous studies (Bentley & Bossé, 2018; van Steenbrugge et al., 2014). Utilizing multiple models and manipulative tools to introduce fractions facilitates a clearer understanding of the concept, making it more accessible for students. This approach helps to concretize fractions for elementary school students, who are still operating at a concrete level of understanding (Kieren, 1976).

Apart from that, it turns out that pre-service teachers failed (were unsuccessful) in representing numerical transitions for the mixed fraction model compared to other ordinary fraction transitions in both types of addition and subtraction operations. This situation is in line with several other studies, mentioning that pre-service teachers usually experience conceptual misunderstandings (misconceptions) of operational procedures for adding and subtracting fractions (Bentley & Bossé, 2018; Lee & Boyadzhiev, 2020). In numerical representations, students face greater challenges when performing subtraction operations, often making mistakes in the procedural steps and in writing the numerical expressions. Notably, in the case of fractions with different denominators, pre-service teachers tended to maintain their instinctual habits, treating the denominator and numerator as distinct entities and

simply adding them as separate numbers. Such issues have been extensively documented in the literature (e.g. Carpenter et al., 1988; Kar & Işık, 2014).

Another finding was that pre-service teachers had difficulty conceptually solving problems and transition procedures for adding and subtracting fractions, especially in determining the numerator and denominator, and simplifying the numerator of the fractions. This finding is corroborated by those of previous studies which indicated that pre-service teachers had poor abilities in determining the numerator and denominator in mixed fractions with addition and subtraction operations (Vula & Kingji-Kastrati, 2018; Reeder & Utley, 2017). Therefore, it can be concluded that while pre-service teachers demonstrated strong operational skills, they did not fully grasp the conceptual meaning of fractions, particularly in understanding the representation of fractions in terms of the part-whole relationship. (Kolar et al., 2018). Likewise determined that pre-service teachers were able to perform division in rational numbers algorithmically/operationally but had difficulty expressing it conceptually. This result is in accordance with the findings of (e.g. Abdullah, 2014; Kara & Incikabi, 2018; Switzer, 2018; Lee & Lee, 2023) suggesting that students experience difficulty in transitioning between different rational number representations.

The current study also revealed that pre-service teachers had difficulty understanding the concept of interpretation procedures in determining the suitability of translations for visual representations with fractions. These skills are related

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to the skills of determining signs on a number line, shading model representations, and/or translating images represented by fraction representations for addition and subtraction. This finding is in line with previous research indicating that pre-service teachers faced problems in interpreting the concept of fractions represented by visual representations (Barbosa & Vale, 2021). Conversely, pre-service teachers were more proficient in generating symbolic representations than in other types. However, when they illustrated division in fractions using multiple representations, they often made the mistake of merely presenting the results on a model or number line instead of accurately modeling the operation. Another common error involved displaying only the numerical values of the numerator and denominator in their representations without fully utilizing the models. These mistakes indicate that the pre-service teachers overlooked the conceptual aspects of division in general when demonstrating fractions through various representations. Researchers (Adu-gyamfi, 2019; Bentley & Bossé, 2018; Lee & Boyadzhiev, 2020). It can be argued that many misunderstandings related to fractions originate from the tendency to apply learned behaviors from natural numbers to fractions.

## CONCLUSIONS AND SUGGESTIONS

This research focused on describing pre-service teachers' ability to create multiple representations (numerical, models, number lines) from addition-subtraction operations on fractions and analyzing the teachers' performance in constructing each type

of representation. The findings of this research suggest that teachers' misconceptions regarding multiple representations can impede students' understanding of mathematical operations involving rational expressions, exponents, and various other algebraic processes.

Common errors and misconceptions related to fraction operations are caused by students' reliance on memorizing mathematical procedures without conceptual understanding. In this scenario, the pre-service teachers did not fully understand the nature of fractions and the meaning of fraction operations. Instead, they often recalled what they had learned about fractions, for example "finding the same denominator", or "vice versa" without understanding why and what the problem of fraction representation means. The pre-service teachers' work in this study showed that these teachers would automatically look for common denominators without understanding the problem, rewrite division problems as multiplication, or use cross multiplication incorrectly.

In conclusion, Pre-service mathematics teachers need to enhance their skills in creating multiple representations to foster conceptual understanding among their future students. Therefore, it is crucial to emphasize the significance of utilizing multiple representations in the training programs offered to prospective mathematics teachers during their higher education studies. Research needs to be carried out on how to improve the ability of prospective mathematics teachers to use multiple representations.

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